GCE MARKING SCHEME

SUMMER 2016

Mathematics – M3
0982/01

INTRODUCTION

This marking scheme was used by WJEC for the Summer 2016 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conference, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about this marking scheme.

GCE Mathematics - M3

Summer 2016 Mark Scheme

Q	Solution	Mark	Notes
1(a)	N2L applied to particle		
	$1800 - 120v = 60a$	M1	dim correct equation
	Divide by 60 and $a = \frac{\mathrm{d}v}{\mathrm{d}t}$		
	$\frac{\mathrm{d}v}{\mathrm{d}t} = 30 - 2v$	A1	convincing
1(b)	$\int \frac{dv}{30-2v} = \int dt$	M1	correct sep. of variables
	$-\frac{1}{2}\ln\|30-2v\| = t$ (+C)	A1A1	A1 for $\ln\|30-2v\|$
			A2 all correct, any form.
	When $t = 0$, $v = 8$	m1	initial conditions used
	$C = -\frac{1}{2}\ln 14$		
	$t = \frac{1}{2}\ln\left\|\frac{14}{30-2v}\right\|$		
	$e^{2t} = \frac{14}{30-2v}$	m1	correct inversion at any stage ft similar expression
	$30 - 2v = 14e^{-2t}$		
	$v = 15 - 7e^{-2t}$	A1	any correct simplified expression
	Limiting value of v = <u>15</u>	B1	cao. Allow if e^{-kt}, $k>0$.

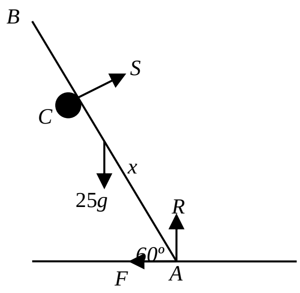
Q	Solution	Mark	Notes
2(a).	$x = A\sin\omega t + B\cos\omega t$.		
	$\frac{dx}{dt} = v = A\omega\cos\omega t - B\omega\sin\omega t$.	B1	
	$\frac{d^2x}{dt^2} = -A\omega^2\sin\omega t - B\omega^2\cos\omega t$	M1	
	Hence,		
	$\frac{d^2x}{dt^2} = -\omega^2 x$	A1	convincing
	Therefore motion is SHM		
	Value of x at centre of motion = 0	B1	
	Amplitude a = value of x when v = 0		
	$A\omega\cos\omega t - B\omega\sin\omega t = 0$	M1	
	$\tan\omega t = \frac{A}{B}$		
	$\sin\omega t = \frac{A}{\sqrt{A^2+B^2}}$ $\cos\omega t = \frac{B}{\sqrt{A^2+B^2}}$	m1	either expression
	$a = A\frac{A}{\sqrt{A^2+B^2}} + B\frac{B}{\sqrt{A^2+B^2}}$		
	$a = \sqrt{A^2+B^2}$	A1	cao
2(b)(i)	using $v^2 = \omega^2(a^2 - x^2)$	M1	
	$25 = \omega^2(a^2 - 25)$		
	$169 = \omega^2(a^2 - 9)$	A1	either equation correct
	Subtract		
	$144 = 16\omega^2$	m1	oe
	$\omega = 3$		
	Amplitude = a		
	$25 = 3^2(a^2 - 25)$	m1	substitution
	Period $= \frac{2\pi}{\omega} = \frac{2\pi}{3}$	A1	cao
	$a^2 = \frac{250}{9}$, $a = \frac{5\sqrt{10}}{3}$ = <u>5.27 (m)</u>	A1	cao
2(b)(ii)	$x = \frac{5\sqrt{10}}{3}\sin(3t)$	M1	accept sin/cos, a, ω
	$x = \frac{5\sqrt{10}}{3}\sin(3\times 0.3)$	A1	ft derived a, ω
	x = <u>4.128 (m)</u>	A1	cao

Q	Solution	Mark	Notes
	Alternative solution		
2(a).	$x = A\sin\omega t + B\cos\omega t$.		
	$x = R\sin(\omega t + \varepsilon)$	M1	
	$A\sin\omega t + B\cos\omega t$		
	$= R\sin\omega t\cos\varepsilon + R\cos\omega t\sin\varepsilon$	m1	si
	$R\cos\varepsilon = A$		
	$R\sin\varepsilon = B$		
	$R = \sqrt{A^2 + B^2}$	A1	
	$\varepsilon = \tan^{-1}\left(\frac{B}{A}\right)$	A1	
	$x = \sqrt{A^2 + B^2}\sin(\omega t + \tan^{-1}\left(\frac{B}{A}\right))$		
	Therefore motion is SHM	A1	
	Value of x at centre of motion = 0	B1	
	Amplitude = $\sqrt{A^2 + B^2}$	A1	

Q	Solution	Mark	Notes
3	Auxiliary equation		
	$m^2 + 6m + 9 = 0$	M1	
	$(m + 3)^2 = 0$		
	$m = -3$ (twice)	A1	
	CF is $x = (A + Bt)e^{-3t}$	B1	ft values of m
	For PI, try $x = at + b$	M1	
	$\frac{dx}{dt} = a$		
	$\frac{d^2x}{dt^2} = 0$		
	$6a + 9(at + b) = 27t$	A1	
	Comparing coefficients	m1	
	$9a = 27$		
	$a = 3$		
	$18 + 9b = 0$		
	$b = -2$	A1	both values
	General solution is		
	$x = (A + Bt)e^{-3t} + 3t - 2$		
	When $t = 0$, $x = 0$	m1	used
	$0 = A - 2$		
	$A = 2$	A1	cao
	$\frac{dx}{dt} = -3(A + Bt)\,e^{-3t} + Be^{-3t} + 3$	B1	ft similar expressions
	When $t = 0$, $\frac{dx}{dt} = 0$,		
	$0 = -3A + B + 3$		
	$B = 3$	A1	ft similar expressions
	$x = (2 + 3t)e^{-3t} + 3t - 2$		
	When $t = 2$		
	$x = 8e^{-6} + 4$	A1	cao
	$x = \underline{4.(02)\ (4.01983)}$		

Q	Solution	Mark	Notes
4(a).	Use of N2L $8g - 0.4v^2 = 8a$	M1	
	$196 - v^2 = 20v\dfrac{\mathrm{d}v}{\mathrm{d}x}$	A1	use of $a = v\dfrac{\mathrm{d}v}{\mathrm{d}x}$,convincing
4(b)	$\int \mathrm{d}x = \int \dfrac{20v\mathrm{d}v}{196 - v^2}$	M1	correct sep variables
	x (+C) = $20 \times -\dfrac{1}{2}\ln\lvert 196 - v^2 \rvert$	A1A1	A1 for $\ln\lvert 196 - v^2 \rvert$, A1 all correct
	x (+C) = $-10\ln\lvert 196 - v^2 \rvert$		
	When $x = 0$, $v = 0$	m1	
	C = -10ln196		
	$x = 10\ln\left\lvert \dfrac{196}{196 - v^2} \right\rvert$	A1	cao
	When $v = 10$		
	$x = 10\ln\dfrac{196}{96} = \underline{7.14\ (\text{m})}$	A1	cao
4(c)	$196 - v^2 = 20\dfrac{\mathrm{d}v}{\mathrm{d}t}$		
	$\int \mathrm{d}t = \int \dfrac{20dv}{14^2 - v^2}$	M1	correct sep variables
	$t = \dfrac{20}{2 \times 14}\ln\left\lvert \dfrac{14 + v}{14 - v} \right\rvert$ +(C)	A1A1	A1 for $\ln\left\lvert \dfrac{14+v}{14-v} \right\rvert$, A1 all correct
	When $t = 0$, $v = 0$	m1	used
	C = 0	A1	
	$t = \dfrac{5}{7}\ln\left\lvert \dfrac{14 + v}{14 - v} \right\rvert$		
	$e^{1.4t} = \dfrac{14 + v}{14 - v}$	m1	inversion
	$v = 14\left(\dfrac{e^{1\cdot 4t} - 1}{e^{1\cdot 4t} + 1}\right)$	A1	cao any correct expres.
	When $t = 2$		
	$v = \underline{12.39}$	A1	cao

Q	Solution	Mark	Notes
5	Speed of *A* just before string becomes taut is given by		
	$v^2 = u^2 + 2as$, a=(±)9.8, s=(1.8-0.2)	M1	
	$v^2 = 0+2\times9.8\times1.6$		
	$v = 5.6$ (ms^{-1})	A1	
	Impulse = change in momentum	M1	used
	Apply to *A*		
	$J = 2 \times 5.6 - 2v$	A1	ft answer in (a)
	Apply to *B*		
	$J = 5v$	B1	
	Solving simultaneously	m1	
	$2 \times 5.6 - 2v = 5v$		
	$7v = 11.2$		
	Speed of *B* = <u>1.6 (ms^{-1})</u>	A1	cao
	$J = 5v =$ <u>8 (Ns)</u>	A1	ft speed of *B*

Q	Solution	Mark	Notes
6(a)		A2	-1 each error
6(b)	Resolve vertically	M1	equation, no missing, no extra force. sin/cos
	$S\cos60^{\circ} + R = 25g$	A1	
	Resolve horizontally	M1	equation, no missing no extra force. sin/cos
	$F = S\sin60^{\circ}$	A1	
	$F = 0.3R$	B1	used
	$0.3R = S\sin60^{\circ}$		
	$R = \frac{\sqrt{3}}{2\times0\cdot3}S$		
	$0.5S + R = 25g$		
	$0.5S + \frac{\sqrt{3}}{2\times0\cdot3}S = 25\times9.8$	m1	eliminating one variable
			Depends on both M's
	$S = \underline{72.34\text{ (N)}}$	A1	cao
	$R = \underline{208.83\text{ (N)}}$	A1	cao
6(c)	Moments about A	M1	equation, no missing, no extra force. dim correct
	$Sx = 25g\times5\cos60^{\circ}$	A1	LHS correct
		A1	RHS correct
	$x = \frac{25\times9\cdot8\times5\times\cos60^{\circ}}{72\cdot340711}$		
	$x = \underline{8.46(69)}$	A1	cao