GCE MARKING SCHEME

MATHEMATICS - M1-M3 & S1-S3
AS/Advanced

SUMMER 2015

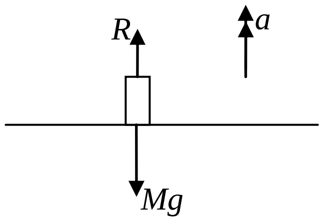
INTRODUCTION

The marking schemes which follow were those used by WJEC for the Summer 2015 examination in GCE MATHEMATICS M1-M3 & S1-S3. They were finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conferences were held shortly after the papers were taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conferences was to ensure that the marking schemes were interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conferences, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about these marking schemes.

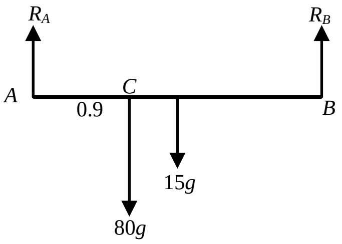
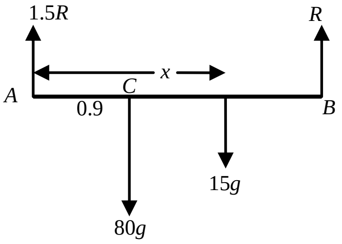
M1

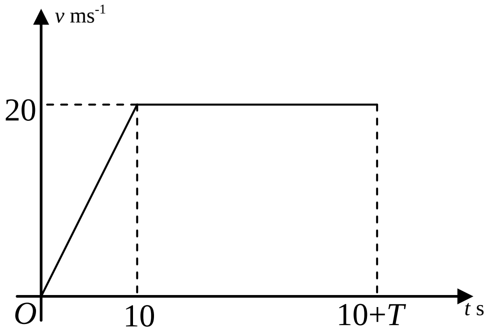
Q	Solution	Mark	Notes
1.			
	N2L applied to man	M1	R and Mg opposing. dim correct
	$R - Mg = Ma$	A1	
	$680 = M(9.8 + 0.2)$		
	$M = \underline{68}$	A1	cao
	N2L applied to Lift and Man	M1	T and weight opposing. dim correct.
	$T - 1868g = 1868a$	A1	ft M
	$T = \underline{18680\ \text{(N)}}$	A1	ft M

Q	Solution	Mark	Notes
2.	Apply N2L to *B*	M1	dim correct, all forces. allow 5*a* RHS
	$5g - T = 0$	A1	$5g$ and T opposing.
	Resolve perpendicular to plane for *A*	M1	allow sin
	$R = 4g\cos\alpha$	A1	
	Apply N2L to *A*	M1	Friction opposes motion. Allow 4*a* RHS and/or cos
	$T - 4g\sin\alpha - F = 0$	A1	
	At limiting equilibrium $F = \mu R$	M1	used
	$\mu = \frac{F}{R} = \frac{45g}{48g} = \frac{15}{16}$	A1	convincing
	$T = 5g = 49$		
	$F = T - 4g\sin\alpha = \frac{45g}{13} = \frac{441}{13} = 33.9231$		
	$R = 4g \times \frac{12}{13} = \frac{48g}{13} = \frac{2352}{65} = 36.1846$		

Q	Solution	Mark	Notes
3(a)	Conservation of momentum	M1	attempted, equation, dim correct.
	$3\times8 + 5\times2 = 3v_A + 5\,v_B$	A1	
	$3v_A + 5\,v_B = 34$		
	Restitution	M1	
	$v_B - v_A = -\frac{1}{3}(2 - 8)$	A1	
	$v_B - v_A = 2$		
	$3v_A + 5\,v_B = 34$ $-3v_A + 3v_B = 6$		
Adding		m1	dep on both M's
	$8v_B = 40$		
	$v_B = \underline{5\ (\text{ms}^{-1})}$	A1	cao
	$v_A = \underline{3(\text{ms}^{-1})}$	A1	cao
3(b)	Impulse = change of momentum	M1	used
	$I = 5 \times 5 - 5 \times 2 = \underline{15\ (\text{Ns})}$	A1	ft v_A or v_B

Q	Solution	Mark	Notes
4	Moments about *x*-axis		
	$=5\times(-1) + 2\times(3) + 3\times5 + 6\times0$	B1	
	$16y = 16$	M1	si
	$y = 1$	A1	cao
	Moments about *y*-axis		
	$=5\times4 + 2\times2 + 3\times(-2) + 6\times(-3)$	B1	
	$16x = 0$	M1	si
	$x = 0$	A1	cao

Q	Solution	Mark	Notes
5(a)			
			
	Moments about A	M1	3 terms, dim correct Equation required
	$2.8R_B = 80g\times0.9 + 15g\times1.4$	A1 B1	correct equation any correct moment
	$R_B = \underline{325.5\ (\text{N})}$	A1	cao
	Vertical forces in equilibrium	M1	all forces, no extra
	$R_A + R_B = 80g + 15g$	A1	
	$R_A = \underline{605.5\ (\text{N})}$	A1	cao
5(b)			
			
	Resolve vertically	M1	
	$1.5R + R = 95g$	A1	
	$R = 38g$		
	Moments about A	M1	3 terms, dim correct
	$2.8\times R = 80g\times0.9 + 15g\times x$	A1	oe
	$x = \frac{172}{75} = \underline{2.3\ (\text{m})}$	A1	cao

Q	Solution	Mark	Notes
6(a)			
		B1	labels, units and shape
		B1	(0, 0) to (10, 20)
		B1	(10, 20) to (10+T, 20)
6(b)	$v = u + at$, v=20, u=0, t=10	M1	
	$20 = 0 + 10a$		
	$a = \underline{2\ (\text{ms}^{-2})}$	A1	
6(c)	Total distance = area under graph	M1	attempted
	$D = 0.5 \times 10 \times 20\ +\ 20T$	B1	one correct area
	$D = 100 + 20T$ (m)	A1	cao
6(d)	$s = ut + 0.5at^2$, u=0, t=5+T, a=2	M1	
	$s = 0.5 \times 2 \times (5+T)^2$	A1	
	$D = 25 + 10T + T^2$		
	$25 + 10T + T^2 = 100 + 20T$	M1	Ft exp for D in (d) and (c)
	$T^2 - 10T - 75 = 0$		
	$(T + 5)(T - 15) = 0$		
	$T = 15$	A1	cao
	$D = \underline{400\ (\text{m})}$	A1	cao

Q	Solution	Mark	Notes
7	Resolve in 80 N direction	M1	Equation required
	$80 = P\cos60° + Q\cos45°$	A1	
	Resolve in 25 N direction	M1	Equation required
	$25 = P\sin60° - Q\sin45°$	A1	
	$160 = P + Q\sqrt{2}$		
	$50 = P\sqrt{3} - Q\sqrt{2}$		
Adding		m1	dep on both M's
	$(1 + \sqrt{3})P = 210$		
	$P = \underline{76.9}$	A1	cao
	$Q = \underline{58.8}$	A1	cao
			penalise once if not 1 d.p.

Q	Solution	Mark	Notes
8(a)	Use of $v^2=u^2+2as$ with $u=(\pm)2.1, a=(\pm)9.8$, $s=(\pm)4$.	M1	
	$v^2 = 2.1^2+2\times9.8\times4$	A1	
	$v = 9.1$	A1	allow -
	speed of rebound = $9.1 \times \frac{4}{7}$	m1	
	= $\underline{5.2\ (\text{ms}^{-1})}$	A1	convincing
8(b)	We require smallest n st $\left(\frac{4}{7}\right)^n \times 9.1 < 1$	M1	oe, si trial & error
	4 bounces	A1	

Q	Solution				Mark	Notes
9	*BCD*	45	19	(5)	B1	for 19
	ABDE	160	8	(5)		
	Circle	9π	7	(5)	B1	both 8 and 7 required
	Lamina	205-9π	x	(y)	B1	expressions for areas, oe
	Moments about *AE*				M1	
	$(205-9\pi)x + 9\pi \times 7 = 160 \times 8 + 45 \times 19$				A1	signs correct. Ft table if at least one B1 for c of m gained.
	$x = \underline{10.96}$				A1	cao
	$y = \underline{5}$				B1	

M2

Q	Solution	Mark	Notes
1.	$\mathbf{x}.\mathbf{y} = 0$	M1	used
	$(\sin\theta\ \mathbf{i} + 2\cos 2\theta\ \mathbf{j}).(2\mathbf{i} - \mathbf{j})$ (= 0)	M1	correct method for dot product, no **i**, **j**’s
	$2\sin\theta - 2\cos 2\theta$ (= 0)	A1	
	$\sin\theta - (1 - 2\sin^2\theta) = 0$	m1	$\cos 2\theta = 1 - 2\sin^2\theta$ depends on both M’s
	$2\sin^2\theta + \sin\theta - 1 = 0$		
	$(2\sin\theta - 1)(\sin\theta + 1) = 0$		
	$\sin\theta = 0.5$		
	$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$	A1	both values
	$\sin\theta = -1$		
	$\theta = \frac{3\pi}{2}$	A1	

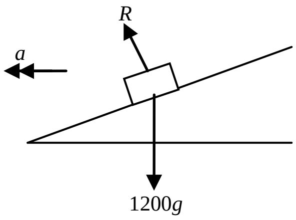
Q	Solution	Mark	Notes
2(a)	Apply N2L to object		
	$1600 - R = 50a$	M1	
	$1600 - kt = 50a$	B1	$R = kt$
	When $t = 2$, $a = -4$	m1	used
	$1600 - 2k = 50 \times (-4)$		
	$k = 900$		
	$1600 - 900t = 50\frac{\mathrm{d}v}{\mathrm{d}t}$		
	$\frac{\mathrm{d}v}{\mathrm{d}t} = 32 - 18t$	A1	convincing
2(b)	$\int dv = \int 32 - 18t\ dt$	M1	increase in power at least once
	$v = 32t - 9t^2 (+ \mathrm{C})$	A1	
	When $t = 2$, $v = 41$	m1	used
	$\mathrm{C} = 9 \times 2^2 - 32 \times 2 + 41$		
	$\mathrm{C} = 13$	A1	cao
	$v = -9t^2 + 32t + 13$		
	When $v = 28$,		
	$28 = -9t^2 + 32t + 13$	m1	substitution of v=28 in c's expression for $v(t)$.
	$9t^2 - 32t + 15 = 0$		
	$(9t - 5)(t - 3) = 0$		
	$t = \frac{5}{9}, 3$	A1	cao

Q	Solution	Mark	Notes
3.			
	N2L	M1	dim correct, all forces
	$T - mg\sin\alpha - R = ma$	A1	correct equation
	$T = \frac{P}{v}$	B1	used si
	$\frac{5P}{16} - 6000\times9.8\times\frac{6}{49} - R = 6000\times2$	A1	correct equation in P & R
	$\frac{5P}{16} - R = 19200$		
	N2L with $a = 0$	M1	dim correct, all forces
	$T - mg\sin\alpha - R = 0$	A1	correct equation
	$\frac{3P}{16} - 6000\times9.8\times\frac{6}{49} - R = 0$	A1	correct equation in P & R
	$\frac{3P}{16} - R = 7200$		
	Solving simultaneously	m1	eliminating one variable, depends on both M's
	$\frac{2P}{16} = 12000$		
	$P = 96000$; $R = 10800$	A1	both answers cao

Q	Solution	Mark	Notes
4(a)	N2L		
	$(4t - 3)\mathbf{i} + (3t^2 - 5t)\mathbf{j} = 0.5\mathbf{a}$	M1	use of **F** = m**a**
	$\mathbf{a} = (8t - 6)\mathbf{i} + (6t^2 - 10t)\mathbf{j}$	A1	cao
	$\mathbf{v} = \int \mathbf{a}\,\mathrm{d}t$	M1	attempted, **i**, **j** retained, power of t increased once
	$\mathbf{v} = (4t^2 - 6t)\mathbf{i} + (2t^3 - 5t^2)\mathbf{j} + (\mathbf{c})$	A1	ft **a** of same diff, not **F**
	When $t = 0$, $\mathbf{v} = 8\mathbf{i} - 7\mathbf{j}$		
	$\mathbf{c} = 8\mathbf{i} - 7\mathbf{j}$	A1	
	$\mathbf{v} = (4t^2 - 6t)\mathbf{i} + (2t^3 - 5t^2)\mathbf{j} + 8\mathbf{i} - 7\mathbf{j}$		
	$\mathbf{v} = (4t^2 - 6t + 8)\mathbf{i} + (2t^3 - 5t^2 - 7)\mathbf{j}$		
4(b)	Impulse = change in momentum	M1	attempted, vector form required
	When $t = 3$, $\mathbf{v} = 26\mathbf{i} + 2\mathbf{j}$	B1	si ft c's **v**
	$0.5(x\mathbf{i} + y\mathbf{j}) - 0.5(26\mathbf{i} + 2\mathbf{j}) = 2\mathbf{i} - 9\mathbf{j}$		
	$(x\mathbf{i} + y\mathbf{j}) = 30\mathbf{i} - 16\mathbf{j}$	A1	cao
	Speed = $\sqrt{30^2 + (-16)^2}$	M1	ft c's x, y
	Speed = $\underline{34 \text{ ms}^{-1}}$	A1	cao

Q	Solution	Mark	Notes
5(a)	$T = 15g$	B1	si
	Hooke's Law		
	$T = \frac{\lambda x}{l} = \frac{1470 \times x}{0 \cdot 4}$	M1	
	$x = \frac{15 \times 9 \cdot 8 \times 0 \cdot 4}{1470}$		
	$x = 0.04$ (m)	A1	cao
5(b)	Let PE be zero at the natural length level.		
	PE = mgh	M1	used
	Initial PE = 15 × 9.8 × (-0.16)	A1	
	Initial PE = -23.52 J		
	Initial EE = $\frac{1}{2} \times \frac{\lambda (x)^2}{l}$	M1	used
	Initial EE = $\frac{1}{2} \times \frac{1470(0 \cdot 16)^2}{0 \cdot 4}$	A1	
	Initial EE = 47.04 J		
	Final KE = $0.5mv^2$		
	Final KE = $7.5v^2$	B1	
	Final PE = 15 × 9.8 × -0.05		
	Final PE = -7.35 J		
	Final EE = $\frac{1}{2} \times \frac{1470(0 \cdot 05)^2}{0 \cdot 4}$		
	Final EE = 4.59375 J		
	Conservation of energy	M1	equation, all 3 types
	$7.5v^2 - 7.35 + 4.59375 = 47.04 - 23.52$	A1	all correct, any form
	$v^2 = 3.5035$		
	v = 1.8718 = 1.87 (ms^{-1})(to 2 d.p.)	A1	

Q	Solution	Mark	Notes
6(a)	Initial $u_{\text{H}} = 35\cos\alpha = (35 \times 0.6 = 21)$ (ms^{-1})	B1	si
	Initial $u_{\text{V}} = 35\sin\alpha = (35 \times 0.8 = 28)$ (ms^{-1})	B1	si
	use of $s= ut+0.5at^2$		
	with s=0,u=28(c),a=(±)9.8	M1	complete method
	$0 = 28t+0.5(-9.8)t^2$	A1	ft u_{V}
	$t(28 - 4.9t) = 0$		
	$t = (0),\ \frac{40}{7}$	A1	
	Total distance travelled by ball = $\frac{40}{7}\times 21$		
	= 120 (m)		
	Ball will not fall into lake.	A1	
6(b)	time to tree = $\frac{17\cdot 5}{21} = \frac{5}{6}$	B1	
	Use $v=u+at$ with u=28(c),a=(±)9.8,t=5/6(c)	M1	oe complete method
	$v = 28 - 9.8 \times \frac{5}{6}$	A1	
	$v = \frac{119}{6}$ (= 19.8333)		
	speed = $\sqrt{\left(\frac{119}{6}\right)^2 + (21)^2}$	m1	
	speed = <u>28.89 (ms^{-1})</u>	A1	cao
	$\theta = \tan^{-1}\left(\frac{119}{6\times 21}\right)$	m1	
	θ = <u>43.36º</u>	A1	cao

Q	Solution	Mark	Notes
7			
	Resolve vertically	M1	equation, dim correct No extra force
	$R\cos12^{o} = 1200g$		
	R = <u>12022.73 (N)</u>	A1	
	N2L towards the centre of motion	M1	dim correct, no extra force
	$R\sin12^{o} = \frac{1200 \times v^{2}}{80}$	A1	
	v = <u>12.91</u>	A1	cao

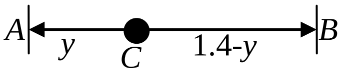
Q	Solution	Mark	Notes
8(a)(i)	Conservation of energy	M1	KE and PE
	$0.5\times3\times5^2$ =		
	$3\times9.8\times0.8(1 - \cos\theta)+ 0.5\times3\times v^2$	A1A1	
	$25 = v^2 + 1.6 \times 9.8 - 1.6 \times 9.8 \cos\theta$		
	$v^2 = \underline{9.32 + 15.68 \cos\theta}$	A1	cao
8(a)(ii)	N2L towards centre of motion	M1	dim correct, 3 terms T, $3g\cos\theta$ opposing
	$T - 3g\cos\theta = \dfrac{3v^2}{0\cdot8}$	A1	
	$T = 3g\cos\theta + 3.75(9.32 + 15.68\cos\theta)$	m1	ft v^2 of form a±bsin/cosθ
	$T = \underline{34.95 + 88.2\cos\theta}$	A1	cao
8(b)	Greatest value of θ occurs when T=0		
	$34.95 + 88.2\cos\theta = 0$	M1	ft $T$of form a±bsin/cosθ
	$\cos\theta = -\dfrac{34\cdot95}{88\cdot2}$		
	$\theta = \underline{113.34^\circ}$	A1	ft a+bcosθ
	Motion stops being circular when θ = 113.34° as string cannot support negative tension. *P* moves under the action of gravity only.	E1	ft θ>90°

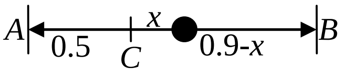
M3

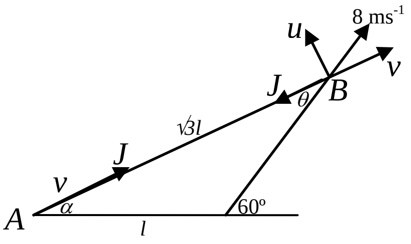
Q	Solution	Mark	Notes
1(a).	Use of N2L $F = 400a$	M1	
	$500\left(\frac{x}{v+2}\right) = 400v\frac{\mathrm{d}v}{\mathrm{d}x}$	A1	use of $a = v\frac{\mathrm{d}v}{\mathrm{d}x}$
	$5x = 4v(v+2)\frac{\mathrm{d}v}{\mathrm{d}x}$		
1(b)(i)	$\int 5x\,\mathrm{d}x = \int 4(v^2 + 2v)\mathrm{d}v$	M1	sep variables
	$\frac{5}{2}x^2 = 4\left(\frac{v^3}{3} + v^2\right) + (C)$	A1A1	
	When $x = 0$, $v = 0$, hence $C = 0$	m1	
	$x = \sqrt{\frac{8}{5}\left(\frac{v^3}{3} + v^2\right)}$	A1	any correct form
1(b)(ii)	When $v = 3$		
	$2.5x^2 = 4(9 + 9)$	m1	
	$x = \frac{12}{\sqrt{5}}$ m = <u>5.37 m</u>	A1	cao
	$a = \frac{5}{4}\left(\frac{12}{5\sqrt{5}}\right)$	m1	substitution of x and v=3.
	$a = \frac{3}{\sqrt{5}}$ = <u>1.34 (ms^{-2})</u>	A1	cao

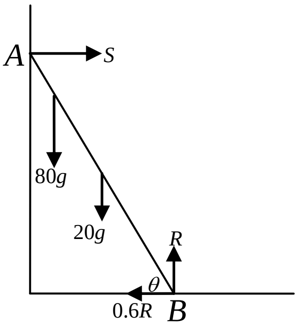
Q	Solution	Mark	Notes
2(a)(i).	N2L $0.5a = -6.5x - 2v$	M1	dimensionally correct
	$\frac{1}{2}\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -\frac{13}{2}x - 2\frac{\mathrm{d}x}{\mathrm{d}t}$	A1	$a=\frac{\mathrm{d}^2x}{\mathrm{d}t^2}$, $v=\frac{\mathrm{d}x}{\mathrm{d}t}$.
	$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 4\frac{\mathrm{d}x}{\mathrm{d}t} + 13x = 0$		
2(a)(ii)	Axilliary equation $m^2 + 4m + 13 = 0$	M1	
	$m = -2 \pm 3i$	A1	
	C. F. is $x = e^{-2t}(A\sin3t + B\cos3t)$	A1	ft m if complex
	When t=0, x=6, $\frac{\mathrm{d}x}{\mathrm{d}t}$=3	m1	used
	$B = 6$		
	$\frac{\mathrm{d}x}{\mathrm{d}t} = -2e^{-2t}(A\sin3t + B\cos3t)$ $+ e^{-2t}(3A\cos3t - 3B\sin3t)$	B1	ft $e^{kt}(A\sin pt + B\cos pt)$
	$-2B + 3A = 3$		
	$A = 5$		
	Solution is $x = e^{-2t}(5\sin3t + 6\cos3t)$	A1	cao
	When t is large, $x \approx 0$	A1	
2(b)	Try PI $x = at + b$	M1	
	$4a + 13(at + b) = 91t + 15$	A1	
	$13a = 91$	m1	equating coefficients
	$a = 7$		
	$4a + 13b = 15$		
	$b = -1$	A1	cao both
	G.S. is $x = e^{-2t}(A\sin3t + B\cos3t) + 7t - 1$		

Q	Solution	Mark	Notes
3(a)	N2L $250a = 250g - 50v$	M1	dimensionally correct
	$5\frac{\mathrm{d}v}{\mathrm{d}t} = 5g - v$	A1	convincing
3(b)	$\int \frac{5dv}{5g-v} = \int dt$	M1	separation of variables
	$-5\ln\lvert 5g - v\rvert = t$ (+C)	A1	correct integration
	When $t = 0$, $v = 0$	m1	used
	$-5\ln\lvert 5g\rvert = \mathrm{C}$	A1	
	$-\frac{t}{5} = \ln\left\lvert\frac{5g-v}{5g}\right\rvert$		
	$5ge^{-\frac{t}{5}} = 5g - v$	m1	correct inversion
	$v = 5g\left(1 - e^{-\frac{t}{5}}\right)$	A1	cao
	When $t = 5$, $v = 5\mathrm{g}(1 - e^{-1})$		
	$= 30.974$ (ms^{-1})	A1	cao numerical answer.
3(c)	$\frac{\mathrm{d}x}{\mathrm{d}t} = 5g - 5ge^{-\frac{t}{5}}$	M1	$v = \frac{\mathrm{d}x}{\mathrm{d}t}$
	$x = 5gt + 25ge^{-\frac{t}{5}}$ (+C)	A1	correct integration ft similar expression
	When $t = 0$, $x = 0$ $\mathrm{C} = -25g$	m1	used
	$x = 5gt + 25ge^{-\frac{t}{5}} - 25g$	A1	
	When $t = 5$, $x = 25\mathrm{g}e^{-1} = 90.13$ (m)	A1	cao

Q	Solution	Mark	Notes
4(a)			
	Tension of spring at *A* = $\frac{15(y-0\cdot3)}{0.3}$	B1	
	Tension of spring at *B* = $\frac{20(1\cdot4-y-0\cdot6)}{0.6}$	B1	
	When in equilibrium $T_A = T_B$	M1	
	$\frac{15(y-0\cdot3)}{0.3} = \frac{20(1\cdot4-y-0\cdot6)}{0.6}$	A1	all correct
	$30y - 9 = 16 - 20y$ $50y = 25$		
	$y = \underline{0.5\text{ (m)}}$	A1	convincing

Q	Solution	Mark	Notes
4(b)(i)			
	$T_A = \dfrac{15(0\cdot2+x)}{0.3}$		
	$T_B = \dfrac{20(0\cdot3-x)}{0.6}$	B1	either
	Force to right = $\dfrac{20(0\cdot3-x)}{0.6} - \dfrac{15(0\cdot2+x)}{0.3}$	M1	allow =/-
	$= -\dfrac{250x}{3}$		
	Apply N2L to P, $7.5\dfrac{\mathrm{d}^2x}{\mathrm{d}t^2} = -\dfrac{250x}{3}$	M1	
	$\dfrac{\mathrm{d}^2x}{\mathrm{d}t^2} = -\dfrac{100}{9}x$		
	Therefore motion is SHM with $\omega=\dfrac{10}{3}$.	A1	si or $\omega^2=100/9$
	Period $= \dfrac{2\pi}{\omega} = \dfrac{3\pi}{5}$	B1	convincing
4(b)(ii)	Amplitude = $\underline{0.25\text{ (m)}}$	B1	
4(b)(iii)	Use $v^2=\omega^2(a^2-x^2)$, $\omega=\dfrac{10}{3}$, $a=0.25, x=0.2$	M1	ft a and ω. oe
	$v^2=(\dfrac{10}{3})^2(0.25^2-0.2^2)$	A1	
	$v = \underline{0.5\text{ (ms}^{-1})}$	A1	cao
4(b)(iv)	$x = a\cos(\omega t)$	M1	oe allow sin/cos, c's a, ω.
	$0.2 = 0.25\cos(\dfrac{10}{3}t)$	A1	
	$t = \dfrac{3}{10}\cos^{-1}(\dfrac{0\cdot2}{0\cdot25})$		
	$t = \underline{0.193\text{ (s)}}$	A1	cao

Q	Solution	Mark	Notes
5.	 		
5(a)	Sine rule		
	$\frac{\sin\theta}{l} = \frac{\sin 120°}{l\sqrt{3}}$	M1	
	$\sin\theta$ = 0.5= 30°	A1	
	α = 60° – 30° = 30°		
5(b)			
	Impulse = change in momentum	M1	used. Allow sin/cos.
	Apply to *B*		
	J = 5×8cos30° - 5v	A1	
	Apply to *A*		
	J = 3v	B1	
	Solving simultaneously	m1	
	$40\frac{\sqrt{3}}{2} - 5v = 3v$		
	Speed of *A* = v = $\frac{5\sqrt{3}}{2}$ = 4.33 (ms^{-1})	A1	cao
	u = 8sin30° = 4 (ms^{-1})	B1	
	Speed of *B* = $\sqrt{4^2 + \left(\frac{5\sqrt{3}}{2}\right)^2}$	m1	
	= 5.9 (ms^{-1})	A1	cao
	J = 3v = 12.99 (Ns)	A1	ft c's 3v

Q	Solution	Mark	Notes
6			
	Resolve vertically	M1	equation, no missing and no extra force.
	$R = 80g + 20g$ (= $100g$)	A1	
	Resolve horizontally	M1	equation, no missing and no extra force.
	$S = 0.6R$ $= 60g = 588$ (N)	A1	
	Moments about B	M1	equation, no missing and no extra force. Dimensionally correct.
	$80g\times5\cos\theta + 20\text{g}\times3\cos\theta = S\times6\sin\theta$ $360\sin\theta = 460\cos\theta$	A2	-1 each error
	$\theta = \tan^{-1}\left(\frac{460}{360}\right) = 51.95^{\circ}$	A1	cao
	The ladder is modelled as a rigid rod.	B1	

S1

Ques	Solution	Mark	Notes
1(a)	$E(X) = 3, \mathrm{Var}(X) = 2.1$ si	**B1**	
	$E(Y) = 2E(X) + 1$	**M1**	
	$= 7$	**A1**	
	$\mathrm{Var}(Y) = 4\mathrm{Var}(X)$	**M1**	
	$= 8.4$	**A1**	
(b)	$P(Y = 7) = P(X = 3)$	**M1**	Award M1 just for this line
	$= \binom{10}{3} \times 0.3^3 \times 0.7^7$	**A1**	Award M0A0 for no working Accept 0.6496 – 0.3828
	$= 0.267$	**A1**	or 0.6172 – 0.3504
2(a)	$P(A \cap B) = P(A) + P(B) - P(A \cup B)$ oe	**M1**	Award B1 for a valid verification
	$P(A \cap B) = 0.4 + 0.5 - 2P(A \cap B)$	**A1**	
	$P(A \cap B) = 0.3$		
(b)	$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$	**M1**	Accept the use of a Venn diagram in (b) and (c)
	$= \frac{0.3}{0.5} = 0.6$	**A1**	
(c)	$P(B \mid A') = \frac{P(B \cap A')}{P(A')} \quad \left(= \frac{P(B) - P(B \cap A)}{1 - P(A)} \right)$	**M1**	
	$= \frac{0.5 - 0.3}{1 - 0.4}$	**A1**	
	$= \frac{1}{3}$ (0.33)	**A1**	
3(a)	P(A chooses G) = 0.3	**B1**	
(b)	P(B chooses Y) = $\frac{8}{10} \times \frac{2}{9} + \frac{2}{10} \times \frac{1}{9}$	**M1A1**	
	$= 0.2$	**A1**	Accept 0.2 without working
(c)	P(Diff colours) = $\frac{3}{10} \times \frac{7}{9} + \frac{5}{10} \times \frac{5}{9} + \frac{2}{10} \times \frac{8}{9}$	**M1A1**	Accept $\frac{^5C_1 \times {^3C_1} + {^5C_1} \times {^2C_1} + {^3C_1} \times {^2C_1}}{^{10}C_2}$
	$= \frac{31}{45}$	**A1**	
4(a)(i)	$P(X = 9) = \frac{e^{-10} \times 10^9}{9!}$	**M1**	Accept 0.4579 – 0.3328 or 0.6672 – 0.5421
	$= 0.1251$	**A1**	Award M0 if no working seen
(ii)	$P(X < 12) = 0.6968$	**M1A1**	Award M1A0 if in adjacent row or column
(b)	Looking at the appropriate section of the table,	**M1**	
	$n = 19$	**A1**	Award M1A0 for 18 or 20

5(a)(i)	P(male and bike) = 0.6×0.75 = 0.45	**M1A1**	
(ii)	P(owns a bike) = $0.6\times0.75+0.4\times0.3$ = 0.57	**M1A1** **A1**	
(b)	P(female|bike) = $\frac{0.12}{0.57}$ = 0.211 (4/19) cao	**B1B1** **B1**	B1 num, B1 denom FT denominator from (a)
6(a) **(i)**	Let X = no. of defective cups so X is B(50,0.05) $P(X=2)=\binom{50}{2}\times0.05^2\times0.95^{48}$ = 0.261	**B1** **M1** **A1**	si Accept 0.5405 – 0.2794 or 0.7206 – 0.4595 M0A0 if no working
(ii)	P($3\le X\le 8$) = 0.9992 – 0.5405 or 0.4595 – 0.0008 = 0.4587	**B1B1** **B1**	Award no marks if no working seen
(b)	Let Y = no. of defective plates so Y is B(250,0.015) ≈ Po(3.75) si P(Y = 4) = $\frac{e^{-3.75}\times3.75^4}{4!}$ = 0.194	**B1** **M1** **A1**	M0A0 if no working
7(a)	$k\left(\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{6}\right)=1$ $k\times\frac{15}{12}=1$ $k=\frac{4}{5}$	**M1** **A1**	Or equivalent Accept verification
(b)	$E(X)=\frac{4}{5}\left(\frac{2}{2}+\frac{3}{3}+\frac{4}{4}+\frac{6}{6}\right)$ = 3.2	**M1** **A1**	
(c)	The possible pairs are (3,4), (4,3), (2,6),(6,2) Prob = $\frac{4}{5}\times\frac{1}{3}\times\frac{4}{5}\times\frac{1}{4}\times2+\frac{4}{5}\times\frac{1}{2}\times\frac{4}{5}\times\frac{1}{6}\times2$ = 0.213 (16/75)	**B1** **M1A1** **A1**	B1 for (3,4),(2,6) M1A0A0 if factor 2 missing

8(a)	P(1st hit with 3rd throw) = 0.7 × 0.7 × 0.3 = 0.147	**M1** **A1**	
(b)(i)	P(F wins 1st throw) = P(G misses) × P(F hits) = 0.8 × 0.3= 0.24	**M1** **A1**	
(ii)	P(F wins with 2nd throw) = P(G miss) × P(F miss) × P(G miss) × P(F hits) = 0.8 × 0.7 × 0.8 × 0.3 = 0.1344	**M1** **A1**	
(iii)	P(F wins) = 0.24 + 0.24 × 0.56 + 0.24 × 0.56^2 + … $=\dfrac{0.24}{1-0.56}$ $=0.545\ \left(\dfrac{6}{11}\right)$	**M1** **B1** **A1**	Award this M1 for realising that the probability is the sum of an infinite geometric series
9(a)	$E\left(\dfrac{1}{X}\right)=\dfrac{4}{9}\int_1^2\dfrac{1}{x}(4x-x^3)\mathrm{d}x$ $=\dfrac{4}{9}\left[4x-\dfrac{x^3}{3}\right]_1^2$ = 0.741 (20/27)	**M1A1** **A1** **A1**	M1 for the integral of $\dfrac{1}{x}f(x)$ A1 for completely correct although limits may be left until 2nd line Award M0 if no working
(b)(i)	$F(x)=\dfrac{4}{9}\int_1^x(4u-u^3)\mathrm{d}u$ $=\dfrac{4}{9}\left[2u^2-\dfrac{u^4}{4}\right]_1^x$ $=\dfrac{8x^2}{9}-\dfrac{x^4}{9}-\dfrac{7}{9}$	**M1** **A1** **A1**	Allow *x* as dummy variable Limits may be left until next line but must then be applied
(ii)	P(1.25 ≤ *X* ≤ 1.75) = F(1.75) – F(1.25) = 0.5625 (9/16)	**M1** **A1**	FT from (b)(i) if possible
(iii)	The median *m* satisfies $\dfrac{8m^2-m^4-7}{9}=0.5$ $m^4-8m^2+11.5=0$ $m^2=\left(\dfrac{8\pm\sqrt{64-46}}{2}\right)=1.88$ $m=1.37$	**M1** **A1** **A1** **A1**	FT from (b)(i) if possible Condone the absence of ±

S2

Ques	Solution	Mark	Notes
1(a)	$H_0 : \mu = 120; H_1 : \mu \neq 120$	**B1**	
(b)	$\bar{x} = \frac{\Sigma x}{10}$	**M1**	
	$= 119.2$	**A1**	
	Test statistic $= \frac{119.2 - 120}{\sqrt{1.2^2/10}}$	**M1A1**	Award M0 if 10 omitted
	$= -2.11$	**A1**	
	Value from tables = 0.01743	**A1**	
	p-value = 0.03486	**A1**	FT from line above
	Strong evidence that the mean speed has changed.	**B1**	Accept ‘mean speed has decreased’ FT the p-value if less than 0.05
2(a)	95th percentile = 82 + 1.645 × 2.5	**M1**	
	= 86.1	**A1**	
(b)	Let X=weight of a man, Y=weight of a woman		
	$z_1 = \frac{68-65}{2} = 1.5$	**M1A1**	M0 if no working
	$z_2 = \frac{64-65}{2} = -0.5$	**A1**	
	$P(Y < 1.5) = 0.9332$ or $P(Y > -0.5) = 0.6915$	**A1**	
	$P(Y < -0.5) = 0.3085$ or $P(Y > 1.5) = 0.0668$	**A1**	
	$P(64 < Y < 68) = 0.6247$	**A1**	
(c)	Let $U = \sum_{i=1}^{3} X_i + \sum_{i=1}^{4} Y_i$		
	E(U) = 3 × 82 + 4 × 65 = 506	**B1**	
	Var(U) = $3 \times 2.5^2 + 4 \times 2^2 = 34.75$	**M1A1**	
	$z = \frac{500-506}{\sqrt{34.75}} = -1.02$	**M1A1**	
	Prob = 0.8461	**A1**	
3(a)	Let X, Y = measured sugar contents of A,B		
	($\sum x = 1612; \sum y = 1584$)		
	$\bar{x} = 201.5; \bar{y} = 198$	**B1B1**	
	SE of diff of means= $\sqrt{\frac{1.5^2}{8} + \frac{1.5^2}{8}}$ (0.75)	**M1A1**	M0 if 8 omitted or only one term
	99% confidence limits for the difference are $201.5 - 198 \pm 2.5758 \times 0.75$	**m1A1**	Award this A1 for z if m1 given
	[1.57, 5.43]	**A1**	
(b)	$4.81 - 2.19 = 2z \times 0.75$	**M1A1**	FT from (a)
	z = 1.75	**A1**	
	Confidence level = 92%	**A1**	

4(a)	Under H_0, X is B(20,0.4) si $\begin{pmatrix} P(X \ge 13) = 0.0210 \\ P(X \ge 14) = 0.0065 \end{pmatrix}$ $X \ge 14$ has significance level closest to 1%	**B1** **M1** **A1**	Award M1 for valid attempt at using tables Award M1A0 for 13 or 15
(b)	Let Y = number of hits Under H_0, Y is B(120, 0.4) $\approx$ N(48, 28.8) si Test statistic = $\frac{54.5-48}{\sqrt{28.8}}$ = 1.21 p-value = 0.1131 Insufficient evidence to conclude that his shooting has improved	**B1** **B1** **M1A1** **A1** **A1** **B1**	Award M1A0 for incorrect or no continuity correction but FT for following marks No cc gives z = 1.30, p = 0.0968 Wrong cc z = 1.40, p = 0.0808 FT the p-value
5	Let X = score on a single die. Then E(X) = 3.5 and Var(X) = $\frac{91}{6} - 3.5^2 = \frac{35}{12}$ Let Y = mean of scores on 100 dice. Then by the Central Limit Theorem, $Y \approx$ N(3.5, 35/1200). $z = \frac{3.75-3.5}{\sqrt{35/1200}}$ = (±)1.46 Prob = 0.0721	**B1** **M1A1** **M1A1** **m1A1** **A1** **A1**	FT their mean and variance Use of continuity correction gives z = 1.43, p = 0.0764
6(a)(i) **(ii)**	$H_0 : \mu = 1.2; H_1 : \mu < 1.2$ Under H_0, X is Po(12) si $P(X \le 9)$ = 0.2424 Insufficient evidence to conclude that the (mean) number of breakdowns has decreased.	**B1** **B1** **M1** **A1** **B1**	Accept 12 in place of 1.2 FT the p-value
(b)	Under H_0, Y is Po(120) $\approx$ N(120,120) $z = \frac{101.5-120}{\sqrt{120}}$ $= -1.69$ p-value = 0.0455 Strong evidence to conclude that the (mean) number of breakdowns has decreased.	**B1** **M1A1** **A1** **A1** **B1**	Award M1A0 for incorrect or no continuity correction but FT for following marks No cc gives z = – 1.73, p=0.0418 Wrong cc, z = – 1.78, p = 0.0375 FT the p-value if less than 0.05

7(a)(i)	$P(Y \le y) = P(\sqrt{X} \le y)$	**M1**	
	$= P(X \le y^2)$	**A1**	
	$= \dfrac{y^2 - a}{b - a}$	**A1**	
(ii)	Attempting to differentiate,	**M1**	
	giving $\dfrac{2y}{b-a}$	**A1**	
	$f(y) = \dfrac{2y}{b-a}$ for $\sqrt{a} \le y \le \sqrt{b}$ = 0 otherwise	**A1**	
(b)	We are given that		
	$\dfrac{a+b}{2} = 5.5$ and $\dfrac{(b-a)^2}{12} = 3$	**B1B1**	
	Solving, *a* = 2.5, *b* = 8.5	**M1** **A1A1**	

S3

<table>
<tr><th>Ques</th><th>Solution</th><th>Mark</th><th>Notes</th></tr>
<tr>
<td>1</td>
<td>
The sample space is as follows.

EITHER
<table>
<tr><th>Samples</th><th><i>R</i></th><th><i>M</i></th></tr>
<tr><td>1,2,2</td><td>1</td><td>2</td></tr>
<tr><td>1,2,4</td><td>3</td><td>2</td></tr>
<tr><td>1,2,6</td><td>5</td><td>2</td></tr>
<tr><td>1,2,6</td><td>5</td><td>2</td></tr>
<tr><td>1,2,4</td><td>3</td><td>2</td></tr>
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<tr><td>1,4,6</td><td>5</td><td>4</td></tr>
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<tr><td>1,6,6</td><td>5</td><td>6</td></tr>
<tr><td>2,2,4</td><td>2</td><td>2</td></tr>
<tr><td>2,2,6</td><td>4</td><td>2</td></tr>
<tr><td>2,2,6</td><td>4</td><td>2</td></tr>
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<tr><td>2,4,6</td><td>4</td><td>4</td></tr>
<tr><td>2,4,6</td><td>4</td><td>4</td></tr>
<tr><td>2,6,6</td><td>4</td><td>6</td></tr>
<tr><td>4,6,6</td><td>2</td><td>6</td></tr>
</table>
OR
<table>
<tr><th>Samples</th><th><i>R</i></th><th>M</th><th>No. of ways</th></tr>
<tr><td>1,2,2</td><td>1</td><td>2</td><td>1</td></tr>
<tr><td>1,2,4</td><td>3</td><td>2</td><td>2</td></tr>
<tr><td>1,2,6</td><td>5</td><td>2</td><td>4</td></tr>
<tr><td>1,4,6</td><td>5</td><td>4</td><td>2</td></tr>
<tr><td>1,6,6</td><td>5</td><td>6</td><td>1</td></tr>
<tr><td>2,2,4</td><td>2</td><td>2</td><td>1</td></tr>
<tr><td>2,2,6</td><td>4</td><td>2</td><td>2</td></tr>
<tr><td>2,4,6</td><td>4</td><td>4</td><td>4</td></tr>
<tr><td>2,6,6</td><td>4</td><td>6</td><td>2</td></tr>
<tr><td>4,6,6</td><td>2</td><td>6</td><td>1</td></tr>
</table>
</td>
<td>M1
A1
A1
A1

M1
A1
A1
A1</td>
<td>A1 for the samples column
A1 for the R column
A1 for the M column
Minus A1 if 1 or 2 rows omitted
Minus A2 if 3 or 4 rows omitted

A1 for columns 1 and 4
A1 for the R column
A1 for the M column
Minus A1 if 1 or 2 rows omitted
Minus A2 if 3 or 4 rows omitted</td>
</tr>
</table>

<table>
<tr>
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<td>The probability distribution of *R* is therefore
<table><tr><td>*r*</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>*P*(*R*=*r*)</td><td>1/20</td><td>2/20</td><td>2/20</td><td>8/20</td><td>7/20</td></tr></table>The probability distribution of *M* is therefore
<table><tr><td>*m*</td><td>2</td><td>4</td><td>6</td></tr><tr><td>*P*(*M*=*m*)</td><td>10/20</td><td>6/20</td><td>4/20</td></tr></table></td>
<td>**M1**
A1
M1
A1</td>
<td>FT for both tables from (a) if sum of probabilities is 1</td>
</tr>
<tr>
<td>**2(a)**
(b)</td>
<td>$\sum x = 192.9; \sum x^2 = 3118.91$
UE of μ = 16.075
UE of $\sigma^2 = \frac{3118.91}{11} - \frac{192.9^2}{132}$
= 1.640
DF = 11 si
Crit value = 3.106
99% confidence limits are
$16.075 \pm 3.106\sqrt{\frac{1.640}{12}}$
giving [14.9,17.2]</td>
<td>**B1B1**
B1
M1
A1
B1
B1
M1
A1
A1</td>
<td>Must be seen
No working need be seen
M0 division by 12
Answer only no marks
FT their s^2 and mean
M0 use of z-values
M0 if 12 omitted
Answer only no marks</td>
</tr>
<tr>
<td>**3(a)**
(b)</td>
<td>$H_0 : \mu_a = \mu_b ; H_1 : \mu_a \neq \mu_b$
SE = $\sqrt{\frac{7.62}{100} + \frac{6.91}{100}}$ (= 0.381…)
Test stat = $\frac{161.17 - 160.53}{0.381}$
= 1.68
Tabular value = 0.04648
p-value = 0.09296
Insufficient evidence to conclude that there is a difference in mean weight.</td>
<td>**B1**
M1A1
M1A1
A1
A1
A1
B1</td>
<td>Treat taking the variances as SDs as a misread, giving
SE = 1.029, Test stat = 0.62, p-value = 0.535
M0 if 100 omitted
FT the p-value</td>
</tr>
<tr>
<td>**4(a)**</td>
<td>$\hat{p} = \frac{54}{90} = 0.6$ si
ESE = $\sqrt{\frac{0.6 \times 0.4}{90}}$ = 0.0516.. si
90% confidence limits are
$0.6 \pm 1.645 \times 0.0516..$
giving [0.515,0.685]</td>
<td>**B1**
M1A1
M1A1
A1</td>
<td></td>
</tr>
</table>

(b)(i)	$\hat{p} = \frac{0.5445 + 0.6485}{2} = 0.5965$	**B1**	Only award if used to find SE in (b)(i)
	$0.6485 - 0.5445 = 2 \times 1.96\sqrt{\frac{0.5965 \times 0.4035}{n}}$	**M1A1**	Award this M1 even if 0.6 used instead of 0.5965
	$n = \left(\frac{2 \times 1.96}{0.104}\right)^2 \times 0.5965 \times 0.4035$	**m1**	
	$n = 342$ cao	**A1**	
(ii)	Number of red squirrels = 342 × 0.5965 = 204	**B1**	FT the *n* from (b)(i)
5(a)	$\sum x = 100, \sum x^2 = 2250,$ $\sum y = 1716.6, \sum xy = 34485$	**B2**	Minus 1 each error
	$S_{xy} = 34485 - 100 \times 1716.6/5 = 153$ si	**B1**	
	$S_{xx} = 2250 - 100^2/5 = 250$ si	**B1**	
	$b = \frac{153}{250} = 0.612$	**M1** **A1**	
	$a = \frac{1716.6 - 0.612 \times 100}{5} = 331.08$	**M1** **A1**	
(b)(i)	SE of $a = \sqrt{\frac{0.25^2 \times 2250}{5 \times 250}}$ (0.3354..)	**M1A1**	FT from (a)
	99% confidence limits are 331.08 ± 2.576 × 0.3354 [330.2, 331.9]	**M1A1** **A1**	
(ii)	SE of $b = \sqrt{\frac{0.25^2}{250}}$ (0.01581…)	**M1A1**	FT from (a)
	Test stat = $\frac{0.612 - 0.65}{0.01581}$	**M1A1**	
	$= -2.40$	**A1**	
	Critical value = 1.96 or *p*-value = 0.0164	**A1**	
	Reject H_0	**A1**	

6(a)(i)	$E(X)=\theta+2\times 2\theta+3\times 3\theta+4(1-6\theta)$	**M1**	
	$=4-10\theta$	**A1**	
	Therefore $E(\overline{X})=4-10\theta$ si	**A1**	
	$E(U)=a(4-10\theta)+b=\theta$ for all θ	**M1**	
	$a=-\frac{1}{10};b=\frac{4}{10}$	**A1**	
	$\left(U=\frac{4}{10}-\frac{1}{10}\overline{X}\right)$		
(ii)	$\text{Var}(X)=\theta+4\times 2\theta+9\times 3\theta+16(1-6\theta)-(4-10\theta)^2$	**M1**	
	$=20\theta(1-5\theta)$	**A1**	
	$\text{Var}(U)=a^2\frac{\text{Var}(X)}{n}$	**M1**	
	$=\frac{\theta(1-5\theta)}{5n}$	**A1**	
(b)(i)	Y is $B(n,1-6\theta)$ so $E(Y)=n(1-6\theta)$	**M1**	
	Therefore $E(V)=cn(1-6\theta)+d=\theta$ (for all θ)	**A1**	
	$c=-\frac{1}{6n};d=\frac{1}{6}$	**A1**	
	$\left(V=\frac{1}{6}-\frac{1}{6n}Y\right)$		
(ii)	$\text{Var}(V)=c^2\text{Var}(Y)=c^2npq$	**M1**	
	$=\frac{\theta(1-6\theta)}{6n}$	**A1**	
(c)	$\frac{\text{Var}(U)}{\text{Var}(V)}=\frac{\theta(1-5\theta)}{5n}\times\frac{6n}{\theta(1-6\theta)}=\frac{6-30\theta}{5-30\theta}$	**B1**	Convincing
	This ratio is greater than 1 so that V is the better estimator.	**B1**	

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