Rewarding Learning

ADVANCED SUBSIDIARY (AS)
General Certificate of Education
2015

Mathematics

Assessment Unit F1

assessing

Module FP1: Further Pure Mathematics 1

AMF11

[AMF11]

WEDNESDAY 24 JUNE, MORNING

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all six** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all six questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 Let the matrix $\mathbf{R} = \begin{pmatrix} 4 & 9 & 0 \\ 0 & -2 & 8 \\ 0 & 0 & 7 \end{pmatrix}$

(i) Calculate $\mathbf{R}\begin{pmatrix} 3 \\ -2 \\ 0 \end{pmatrix}$ [2]

(ii) Explain fully the relationship between the matrix $\mathbf{R}$ and $\begin{pmatrix} 3 \\ -2 \\ 0 \end{pmatrix}$ [2]

(iii) Hence, or otherwise, express $\mathbf{R}^2\begin{pmatrix} 3 \\ -2 \\ 0 \end{pmatrix}$ in the form $\alpha\begin{pmatrix} 3 \\ -2 \\ 0 \end{pmatrix}$, where α is an integer. [4]

2 A circle has equation

$$x^2 + y^2 - 4x - 8y + 10 = 0$$

(i) Find the equation of the tangent to the circle at the point $(-1, 5)$. [5]

(ii) Find the equation of the other tangent to this circle that is parallel to the tangent found in **(i)**. [5]

3 The matrix $\mathbf{M}$ is given by

$$\mathbf{M} = \begin{pmatrix} a & 1 & -2 \\ 1 & 4 & 1 \\ -2 & 1 & a \end{pmatrix}$$

Consider the matrix equation

$$\mathbf{M}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} b \\ 2 \\ 5 \end{pmatrix}$$

(i) Find the values of a for which the matrix equation does not have a unique solution. [5]

(ii) If $a = -2$ and $b = 6$ explain why the matrix equation has no solution. [3]

(iii) If $a = -2$ and $b = 5$ find the general solution of the matrix equation. [4]

4 S is the set of matrices $\begin{pmatrix} r & s \\ s & r \end{pmatrix}$, where r, s are real numbers such that $r^2 \neq s^2$

Prove that S forms a group under matrix multiplication.
You may assume that matrix multiplication is associative. [12]

5 **(a)** Write down the matrix which represents a rotation of 45° anticlockwise about the origin. [2]

(b) The matrix $\mathbf{N} = \begin{pmatrix} 2 & 1 \\ -6 & 3 \end{pmatrix}$

Under the transformation represented by $\mathbf{N}$ the line $y = mx$ is reflected in the y-axis.

Find the possible values of m. [8]

6 **(a)** **(i)** Given that

$$(a + b\mathrm{i})^2 = -5 + 12\mathrm{i}$$

find the real values of a and the corresponding values of b. [8]

(ii) Hence find the complex roots of the quadratic equation

$$z^2 - (4 - \mathrm{i})\,z + (5 - 5\mathrm{i}) = 0$$ [6]

(b) **(i)** Sketch on an Argand diagram the locus of those points w which satisfy

$$|w - (3 + 3\mathrm{i})| = \frac{3}{\sqrt{2}}$$ [3]

(ii) For any point w on the locus described in **(b)(i)**, show that

$$\frac{\pi}{12} \leqslant \arg w \leqslant \frac{5\pi}{12}$$

A solution by scale drawing will not be accepted. [6]

THIS IS THE END OF THE QUESTION PAPER