Rewarding Learning

ADVANCED
General Certificate of Education
2015

Mathematics

Assessment Unit F2

assessing

Module FP2: Further Pure Mathematics 2

AMF21

[AMF21]

TUESDAY 9 JUNE, MORNING

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all eight** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all eight questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 **(i)** Show that

$$\frac{1}{r^2} - \frac{1}{(r+1)^2} \equiv \frac{2r+1}{r^2(r+1)^2}$$ [2]

(ii) Hence or otherwise find

$$\sum_{r=1}^{n} \frac{2r+1}{r^2(r+1)^2}$$ [3]

2 Find, in terms of π, the general solution of the equation

$$\tan 2x + \tan 4x = 0$$ [7]

3 **(i)** Using partial fractions show that

$$\frac{x-4x^2}{(2-x)^2(3+x^2)} \equiv \frac{1}{2-x} - \frac{2}{(2-x)^2} + \frac{x}{3+x^2} \qquad x \neq 2$$ [6]

(ii) Hence or otherwise find the exact value of

$$\int_0^1 \frac{x-4x^2}{(2-x)^2(3+x^2)}\,\mathrm{d}x$$ [5]

4 **(i)** Using Maclaurin's theorem, find the first three terms of the series expansion for

$$f(x) = e^{\tan x}$$ [5]

(ii) Hence write down the first three terms of the expansion for $e^{-\tan x}$ [1]

5 Using the principle of mathematical induction, prove that

$$3^{2n+1} + 2^{n+2}$$

is divisible by 7 for any positive integer n. [6]

6 **(a)** **(i)** Show that the equation of the tangent to the ellipse

$$\frac{x^2}{16} + y^2 = 1$$

at the point P $(4\cos t, \sin t)$ is

$$4y \sin t = -x \cos t + 4$$ [4]

The tangent at P cuts the coordinate axes at Q and R. The midpoint of QR is M.

(ii) As t varies find the Cartesian equation of the locus of the point M. [5]

(b) Show that if the line with equation $y = mx + c$ is to be a tangent to the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

then

$$a^2m^2 + b^2 = c^2$$ [4]

7 A particle P is constrained to move on a fixed line so that at time t seconds its displacement x metres from an origin O is given by the differential equation

$$\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 13x = 40 \cos 3t$$

Given that $x = 2$ and $\frac{dx}{dt} = 13$ when $t = 0$, find the displacement x as a function of t. [13]

8 **(a)** **(i)** Write down the modulus and argument of the complex number $4 + 4i$ [2]

(ii) Solve the equation

$$z^5 = 4 + 4i$$

leaving your answers in exponential form. [6]

(b) Find the smallest positive integer values of p and q so that

$$\frac{\left(\cos\frac{\pi}{8} + i\sin\frac{\pi}{8}\right)^p}{\left(\cos\frac{\pi}{6} - i\sin\frac{\pi}{6}\right)^q} = -1$$

[6]

THIS IS THE END OF THE QUESTION PAPER