A-LEVEL
Mathematics
Paper 1
Mark scheme

Specimen

Version 1.2

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Mark scheme instructions to examiners

General

The mark scheme for each question shows:

- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
dM	mark is dependent on one or more M marks and is for method
R	mark is for reasoning
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
'their'	Indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
sf	significant figure(s)
dp	decimal place(s)

Examiners should consistently apply the following general marking principles

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to students showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the student to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, only the last complete attempt should be awarded marks.

Q	Marking Instructions	AO	Marks	Typical Solution
1	Circles correct answer	AO1.1b	B1	$-\frac{2}{5}$
	Total		**1**	
2	Circles correct answer	AO1.1b	B1	$-\frac{1}{x\sqrt{x}}$
	Total		**1**	
3	Uses either $\cos x \approx 1 - \frac{1}{2}x^2$ or $\sin x \approx x$ (PI)	AO1.2	B1	$\cos 3\theta + \theta \sin 2\theta \approx 1 - \frac{(3\theta)^2}{2} + \theta(2\theta)$ $\approx 1 - \frac{5}{2}\theta^2$
	Substitutes 2θ and 3θ into 'their' expression	AO1.1a	M1	
	Obtains correct answer	AO1.1b	A1	
	Total		**3**	

Q	Marking Instructions	AO	Marks	Typical Solution
4(a)	Demonstrates $p(-3)=0$	AO1.1b	B1	$p(-3)=2(-3)^3+7(-3)^2+2(-3)-3$ $=-54+63-6-3=0$ $p(-3)=0 \Rightarrow x+3$ is a factor
	Constructs rigorous mathematical proof (to achieve this mark, the student must clearly calculate and state that $p(-3)=0$ **and** clearly state that this implies that $x+3$ is a factor)	AO2.1	R1	
(b)	Factorises the numerator and denominator (this mark is achieved for any reasonable attempt at factorisation through the selection of an appropriate method, eg long division)	AO1.1a	M1	$\frac{(x+3)(2x^2+x-1)}{(2x+1)(2x-1)}$ $=\frac{(x+3)(2x-1)(x+1)}{(2x+1)(2x-1)}$ $=\frac{(x+3)(x+1)}{(2x+1)},\ x\neq\pm\frac{1}{2}$
	Finds second factor in numerator or fully factorises denominator (PI by complete factorisation)	AO1.1b	A1	
	Finds fully correct factorised expression (PI by complete factorisation)	AO1.1b	A1	
	Obtains a completely correct solution with restriction on domain stated	AO1.1b	A1	
	Total		**6**	

Q	Marking Instructions	AO	Marks	Typical Solution
5(a)	Recalls $A=\frac{1}{2}r^2\theta$ or $l=r\theta$ PI by use in equation	AO1.2	B1	Area of sector gives $\frac{1}{2}r^2\theta=9$, $\theta=\frac{18}{r^2}$
	Constructs two equations at least one correct	AO1.1a	M1	Perimeter of sector gives $2r+r\theta=15$
	Eliminates θ FT incorrect equations	AO1.1a	M1	$2r+\frac{18}{r}=15$
	Constructs a rigorous mathematical argument to show required result, clearly constructing two correct simultaneous equations and eliminating θ **AG**	AO2.1	R1	$2r^2+18=15r$ $2r^2-15r+18=0$ **(AG)**
(b)	Solves a quadratic equation and finds two values of θ	AO3.1a	M1	$r=\frac{3}{2},r=6$ $r=6\Rightarrow\theta=\frac{1}{2}$ $r=\frac{3}{2}\Rightarrow\theta=8$ $8>2\pi\therefore\theta\neq8$ so only one possible value of θ
	Finds two correct values of r	AO1.1b	B1	
	Finds both values of θ	AO1.1b	A1	
	Gives a valid reason for rejecting one of ‘their’ values	AO2.4	R1	
	Total		**8**	

Q	Marking Instructions	AO	Marks	Typical Solution
6 (a)	Translates rate of change into $\frac{\mathrm{d}m}{\mathrm{d}t}$	AO3.3	M1	$\frac{\mathrm{d}m}{\mathrm{d}t} = -\frac{k}{\sqrt[3]{m}}$
	Translates inverse proportionality by using $\frac{1}{\sqrt[3]{m}}$ in an equation (no need to see minus sign or k to earn this mark)	AO3.3	M1	
	Forms correct equation with correct notation $\frac{\mathrm{d}m}{\mathrm{d}t} = -\frac{k}{\sqrt[3]{m}}$ or equivalent eg $-\frac{\mathrm{d}m}{\mathrm{d}t} = \frac{k}{\sqrt[3]{m}}$ or $\frac{\mathrm{d}m}{\mathrm{d}t} = -km^{-\frac{1}{3}}$	AO1.1b	A1	
(b)	Gives a relevant criticism of the assumption	AO3.5b	E1	Sam's mass is unlikely to follow this model all the time, when he eats his mass will go up. OR Sam's assumption predicts that his mass will decrease indefinitely.
	Total		**4**	
7	Clearly states that equal roots $\Rightarrow b^2 - 4ac = 0$	AO2.4	B1	$\therefore b^2 - 4ac = 0$ for equal roots $k^2 - 4(2k-3)(k-1) = 0$ $7k^2 - 20k + 12 = 0$ $k = \frac{6}{7}, k = 2$
	Forms quadratic expression in k (allow one error)	AO1.1a	M1	
	Obtains correct quadratic equation in k (PI by correct values for k)	AO1.1b	A1	
	Obtains correct values for k for 'their' quadratic equation	AO1.1b	A1F	
	Total		**4**	

Q	Marking Instructions	AO	Marks	Typical Solution
8(a)	States the correct derivative	AO1.1b	B1	$2^x \ln 2$
(b)	Selects an appropriate method for integrating, which could lead to a correct exact solution (this could be indicated by an attempt at a substitution or attempting to write the integrand in the form $f'(x)f(x)^n$)	AO3.1a	M1	Let $u = 2^x$ Then $\frac{du}{dx} = 2^x \ln 2$ And $\frac{1}{\ln 2}\frac{du}{dx} = 2^x$
	Correctly writes integrand in a form which can be integrated (condone missing or incorrect limits)	AO1.1b	A1	$I = \int (3+u)^{\frac{1}{2}} \frac{1}{\ln 2}\frac{du}{dx}\,dx$ $= \frac{1}{\ln 2}\int (3+u)^{\frac{1}{2}}\,du$
	Integrates 'their' expression (allow one error)	AO1.1a	M1	$= \frac{2}{3\ln 2}(3+u)^{\frac{3}{2}} + c$
	Substitutes correct limits corresponding to 'their' method	AO1.1a	M1	Sub limits: $\left[\frac{2}{3\ln 2}(3+u)^{\frac{3}{2}}\right]_1^2$
	Obtains correct value in an exact form	AO1.1b	A1	$\frac{2}{3} \times \frac{1}{\ln 2}\left(5\sqrt{5} - 8\right)$ **ALT (direct inspection)**
	Mark awarded if they have a completely correct solution, which is clear, easy to follow and contains no slips Substitution should be clearly stated in exact form and change of variable or solution by direct inspection should be achieved correctly with correct use of symbols and connecting language	AO2.1	R1	$\int 2^x\sqrt{3+2^x}\,dx$ $= \frac{1}{\ln 2}\int 2^x \ln 2\sqrt{3+2^x}\,dx$ $= \frac{1}{\ln 2}\int 2^x \ln 2(3+2^x)^{\frac{1}{2}}\,dx$ $= \frac{1}{\ln 2} \times \frac{2}{3}(3+2^x)^{\frac{3}{2}}$ $\left[\frac{1}{\ln 2} \times \frac{2}{3}(3+2^x)^{\frac{3}{2}}\right]_0^1$ $\frac{2}{3} \times \frac{1}{\ln 2}(5\sqrt{5} - 8)$
	Total		**7**	

Q	Marking Instructions	AO	Marks	Typical Solution
9(a)(i)	Selects an appropriate routine procedure; evidence of quotient rule or product rule	AO1.1a	M1	$\frac{dy}{dx} = \frac{2(4x^2+7) - 8x(2x+3)}{(4x^2+7)^2}$
	Obtains correct derivative (no need for simplification)	AO1.1b	A1	
(a)(ii)	States clearly that $\frac{dy}{dx} > 0 \Rightarrow y$ is increasing	AO2.4	R1	y is increasing $\Leftrightarrow \frac{dy}{dx} > 0$ $\frac{2(4x^2+7) - 8x(2x+3)}{(4x^2+7)^2} > 0$
	Forms inequality from 'their' $\frac{dy}{dx} > 0$	AO3.1a	B1	$(4x^2+7)^2 > 0$ for all x $\therefore 2(4x^2+7) - 8x(2x+3) > 0$
	Deduces numerator must be positive	AO2.2a	R1	$8x^2 + 14 - 16x^2 - 24x > 0$ $4x^2 + 12x - 7 < 0$ **(AG)**
	Considers denominator alone and sets out clear argument to justify given inequality **AG** Only award this mark if they have a completely correct solution, which is clear, easy to follow and contains no slips	AO2.1	R1	
(b)	Solves the correct quadratic inequality (accept evidence of factorising, completing the square, use of formula, or correct critical values stated)	AO1.1a	M1	$(2x + 7)(2x - 1)$ $x = -\frac{7}{2}, \frac{1}{2}$
	Obtains fully correct answer, given as an inequality or using set notation	AO1.1b	A1	$-\frac{7}{2} < x < \frac{1}{2}$ **Or** $x \in \left(-\frac{7}{2}, \frac{1}{2}\right)$ **Or** $x \in \left[x : -\frac{7}{2} < x < \frac{1}{2}\right]$
	Total		**8**	

Q	Marking Instructions	AO	Marks	Typical Solution
10(a)	Makes a deduction about the lower bound of the function (4)	AO2.2a	B1	
	Correctly states the range of f using set notation	AO2.5	B1	The range of f is the set $\left(x : x > 4, x \in \mathbb{R}\right)$
(b)(i)	States correctly the set they gave in part **(a)**	AO1.2	B1F	$\left(x : x > 4, x \in \mathbb{R}\right)$
(b)(ii)	Interchanges x and y at any stage	AO1.1a	M1	$y = 4 + 3^{-x}$ $x = 4 + 3^{-y}$
	Rearranges and takes logs	AO1.1a	M1	$3^{-y} = x - 4$ $-y = \log_3(x - 4)$
	Obtains correct expression from completely correct working for $\mathrm{f}^{-1}(x)$, notation correct throughout	AO1.1b	A1	$\mathrm{f}^{-1}(x) = -\log_3(x - 4)$
(c)(i)	Obtains $\mathrm{gf}(x)$	AO1.1b	B1	$\mathrm{gf}(x) = \mathrm{g}(4 + 3^{-x})$ $= 5 - (4 + 3^{-x})^{0.5}$
(c)(ii)	Forms equation and rearranges using 'their' $\mathrm{gf}(x) = 2$	AO1.1a	M1	$5 - (4 + 3^{-x})^{0.5} = 2$ $(4 + 3^{-x}) = 9$
	Correctly rearranges to get a single exponential term where logs can be taken. (Follow through provided 'their' equation requires the use of logs.)	AO1.1b	A1F	$3^{-x} = 5$ $x = -\log_3 5$
	Obtains correct solution	AO1.1b	A1	
	Total		**10**	

Q	Marking Instructions	AO	Marks	Typical Solution
11(a)	Completes the square twice or applies standard formula	AO1.1a	M1	$(x+4)^2+(y-6)^2-16-36=12$ $(x+4)^2+(y-6)^2=64$ Centre (–4, 6) Radius = 8
	Obtains correct equation	AO1.1b	A1	
	Obtains correct radius and correct coordinates of *C* Follow through 'their' equation	AO1.1b	A1F	
(b)	Demonstrates a method to find the length *OP* or *OQ* (or their squares), or the coordinates *P* or *Q* using 'their' values from part **(a)**	AO3.1a	M1	$OC^2=4^2+6^2=52$ $OP^2=r^2-OC^2$ $=64-52=12$ $PQ=2OP$ $=2\sqrt{12}=4\sqrt{3}$
	Uses a circle property that may lead to a solution, eg radius and chord meet at right-angles (evidence for this could be the use of Pythagoras or perpendicular gradients)	AO3.1a	M1	
	Finds *OP* or *OQ* or coordinates of *P* or *Q* **CAO**	AO1.1b	A1	
	Obtains length of *PQ* Follow through from 'their' coordinate of *P* and *Q* (Does not need to be in the required form)	AO1.1b	A1F	
	Sets out a well-constructed mathematical argument, using precise statements and correct use of symbols throughout to show the correct required result in required form Only award if they have a completely correct solution, which is clear, easy to follow and contains no slips	AO2.1	R1	
	Total		**8**	

Q	Marking Instructions	AO	Marks	Typical Solution
12	Finds the difference between the maximum and minimum values of y	AO3.1b	M1	$x^2+2xy+2y^2=10$ $2x+2y+2x\frac{dy}{dx}+4y\frac{dy}{dx}=0$ Highest and lowest points occur when $\frac{dy}{dx}=0$ $\frac{dy}{dx}=0 \Rightarrow x=-y$ $y^2-2y^2+2y^2=10$ $y=\pm\sqrt{10}$ ∴Height $=\sqrt{10}-\left(-\sqrt{10}\right)$ $=2\sqrt{10}$ = 6.32 m
	Uses implicit differentiation	AO1.1a	M1	
	Differentiates correctly	AO1.1b	A1	
	States stationary points occur when $\frac{dy}{dx}=0$	AO2.4	R1	
	Uses $\frac{dy}{dx}=0$ to find x in terms of y (or vice versa)	AO1.1a	M1	
	Finds $x=-y$	AO1.1b	A1	
	Deduces maximum and minimum values of y FT 'their' expression provided all M1 marks have been awarded	AO2.2a	A1F	
	States the height of the sculpture above the platform FT 'their' max and min values for y provided all M1 marks have been awarded	AO2.2a	A1F	
	Total		**8**	

Q	Marking Instructions	AO	Marks	Typical Solution
13	Recalls a correct trig identity, which could lead to a correct answer	AO1.2	B1	(LHS $\equiv$) $\cot^2\theta - \cos^2\theta$ $\equiv \dfrac{\cos^2\theta}{\sin^2\theta} - \cos^2\theta$ $\equiv \cos^2\theta\left(\dfrac{1}{\sin^2\theta} - 1\right)$ $\equiv \cos^2\theta\left(\text{cosec}^2\theta - 1\right)$ $\equiv \cos^2\theta\cot^2\theta$ ($\equiv$ RHS) **(AG)**
	Performs some correct algebraic manipulation **and** uses second identity to commence proof (at least two lines of argument)	AO2.1	R1	
	Concludes a rigorous mathematical argument to prove given identity **AG** Must start with one side and through clear logical steps arrive at the other side. In order to be sufficiently clear, each line should be a single step, unless clear further explanation is given.	AO2.1	R1	
	Total		**3**	

Q	Marking Instructions	AO	Marks	Typical Solution
14(a)	Identifies and clearly defines variables.	AO3.1b	B1	Let C = total cost x = length of base edges h = length of height
	Models the cost with an expression of the form $ax^2 + bxh$	AO3.3	M1	$C = 15x^2 + 32xh$
	Eliminates either variable, using volume equation, to form a model for the cost in one variable.	AO3.3	M1	$x^2h = 60$ $h = \dfrac{60}{x^2}$
	Obtains a correct equation to model cost in one variable	AO3.1b	A1	$C = 15x^2 + \dfrac{1920}{x}$ (*)
	Uses their model to find minimum. (at least one term correctly differentiated **and** equated to zero)	AO3.4	M1	Differentiating $30x - \dfrac{1920}{x^2} = 0$ $x^3 = 64$ $x = 4$
	Obtains correct equation	AO1.1b	A1	$h = \dfrac{60}{4^2} = 3.75$
	Obtains correct value for h with correct units in context Award FT from correct substitution into incorrect equation for h but only if all three M1 marks have been awarded, must have correct units.	AO3.2a	A1F	Height of tank is 3.75 m $\dfrac{d^2C}{dx^2} = 30 + \dfrac{3840}{x^3}$ $x = 4 \Rightarrow \dfrac{d^2C}{dx^2} > 0$ therefore minimum **ALT from (*) onwards** $C = 900h^{-1} + 32\sqrt{60}h^{\frac{1}{2}}$ $0 = -900h^{-2} + 16\sqrt{60}h^{-\frac{1}{2}}$
	Performs a correct test of ‘their’ solution: uses the second derivative of ‘their’ expression for *C* in terms of x or h to justify that a minimum value for h has been found OE (Second derivative > 0 or test gradient/values either side)	AO2.4	R1	Height of tank is 3.75 m $\dfrac{d^2C}{dh^2} = 1800h^{-3} - 8\sqrt{60}h^{-\frac{3}{2}}$ $h = 3.75 \Rightarrow \dfrac{d^2C}{dh^2} = 25.6 > 0$ $\therefore$ minimum

Q	Marking Instructions	AO	Marks	Typical Solution
14(b)(i)	Explains that two of the sides of the tank will need to have length $x \pm 0.05$ in order to join them	AO3.5c	E1	The sides will need to overlap to be joined, so two of the side lengths will need to be $x + 0.05$
(b)(ii)	Explains that the refinement is relatively small and unlikely to have a significant effect on the result.	AO3.5a	R1	The minimum cost is likely to increase slightly, but relative to the size of the tank, an extra 5cm is unlikely to make a significant difference.
	Total		**10**	

Q	Marking Instructions	AO	Marks	Typical Solution
15(a)	Separates variables, at least one side correct.	AO3.1a	M1	$3\sqrt{x}\dfrac{\mathrm{d}x}{\mathrm{d}t} = 8\sin 2t$
	Obtains correct separation PI	AO1.1b	A1	$\int 3\sqrt{x}\ \mathrm{d}x = \int 8\sin 2t\ \mathrm{d}t$
	integrates 'their' expressions, at least one of 'their' sides correct	AO1.1a	M1	$\int 3x^{\frac{1}{2}}\ \mathrm{d}x = \int 8\sin 2t\ \mathrm{d}t$
	Obtains correct integral (condone missing + c) CAO	AO1.1b	A1	$2x^{\frac{3}{2}} = -4\cos 2t\,(+c)$
	Substitutes initial conditions, to find + c.	AO3.1b	M1	$2\times(0)^{\frac{3}{2}} = -4\cos(2\times 0) + c$ $c = 4$
	Obtains a correct solution ACF	AO1.1b	A1	$x^{\frac{3}{2}} = 2 - 2\cos 2t$
	Obtains correct solution of the form $x = \mathrm{f}(t)$	AO2.5	A1	$x = \left(2 - 2\cos 2t\right)^{\frac{2}{3}}$
(b)	Obtains correct max height, in cm Award FT from correct substitution into incorrect equation $x = \mathrm{f}(t)$ but only if all three M1 marks have been awarded, must have correct units.	AO3.4	A1F	Max height $= 4^{\frac{2}{3}} = 252$ cm
	Total		**8**	

Q	Marking Instructions	AO	Marks	Typical Solution
16(a)	Identifies zero as number for which student's argument is not true	AO1.2	B1	0
(b)	Uses 'proof by contradiction' Must see commencement of argument including stated assumption and at least two lines of argument	AO2.1	M1	Let a be irrational, and b be a non-zero rational, so $b = \frac{c}{d}$, where $c, d \in \mathbb{Z}; c, d \neq 0$ Assume ab is rational, so $ab = \frac{p}{q}$, where $p, q \in \mathbb{Z}; q \neq 0$ $\therefore \frac{ac}{d} = \frac{p}{q}$ $\therefore a = \frac{pd}{qc}$ so a is rational, which is a contradiction Hence ab must be irrational
	Represents product of rational and irrational numbers in symbolic form	AO2.5	M1	
	Correctly deduces that the product must be irrational	AO2.2a	A1	
	Completes a rigorous mathematical argument, proving that a non-zero rational multiplied by an irrational is irrational Must start with initial assumptions and prove the result convincingly Must define p q c d as integers	AO2.1	R1	
	Total		**5**	

Q	Marking Instructions	AO	Marks	Typical Solution
17	Translates $f'\left(\frac{\pi}{6}\right)$ into $\lim_{h\to 0}\frac{\sin\left(\frac{\pi}{6}+h\right)-\sin\left(\frac{\pi}{6}\right)}{h}$	AO1.1a	M1	$f'\left(\frac{\pi}{6}\right)=\lim_{h\to 0}\left[\frac{\sin\left(\frac{\pi}{6}+h\right)-\sin\left(\frac{\pi}{6}\right)}{h}\right]$
	Uses sin (A + B) identity to replace $\sin\left(\frac{\pi}{6}+h\right)$, to commence argument (at least two lines of argument seen)	AO2.1	M1	$=\lim_{h\to 0}\left[\frac{\sin\frac{\pi}{6}\cos h+\cos\frac{\pi}{6}\sin h-\sin\left(\frac{\pi}{6}\right)}{h}\right]$ $=\lim_{h\to 0}\left[\frac{\frac{1}{2}\cos h+\frac{\sqrt{3}}{2}\sin h-\frac{1}{2}}{h}\right]$
	Obtains correct two term expression involving cos h and sin h	AO1.1b	A1	$=\lim_{h\to 0}\left[\frac{1}{2}\left(\frac{\cos h-1}{h}\right)+\frac{\sqrt{3}}{2}\frac{\sin h}{h}\right]$
	Deduce what happens as $h\to 0$, for one part of 'their' expression using the limit of $\frac{\sin h}{h}$ OR by using small angles approximations	AO2.2a	R1	$=\lim_{h\to 0}\left[\frac{1}{2}\left(\frac{-2\sin^2\left(\frac{h}{2}\right)}{\frac{2h}{2}}\right)+\frac{\sqrt{3}}{2}\frac{\sin h}{h}\right]$ $=\left(\lim_{h\to 0}\frac{-\sin\left(\frac{h}{2}\right)}{2}\right)\left(\lim_{h\to 0}\frac{\sin\left(\frac{h}{2}\right)}{\left(\frac{h}{2}\right)}\right)+\frac{\sqrt{3}}{2}\lim_{h\to 0}\frac{\sin h}{h}$
	Deduce what happens as $h\to 0$, for the second part of 'their' expression using the limit of (cos h – 1)h OR by using small angle approximations	AO2.2a	R1	$=0\times 1+\frac{\sqrt{3}}{2}\times 1$ $=\frac{\sqrt{3}}{2}$
	Completes a rigorous argument leading to the correct exact value, with all the steps in the method clearly shown.	AO2.1	R1	
	Total		**6**	
	TOTAL		**100**	