Write your name here

Surname	Other names

Pearson Edexcel GCE

Centre Number	Candidate Number

Mechanics M5

Advanced/Advanced Subsidiary

Tuesday 19 June 2018 – Afternoon
Time: 1 hour 30 minutes

Paper Reference
6681/01

You must have:
Mathematical Formulae and Statistical Tables (Pink)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}, and give your answer to either two significant figures or three significant figures.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

Pearson

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1. A small bead is threaded on a smooth straight horizontal wire. The wire is modelled as a line with vector equation $\mathbf{r} = (2 + \lambda)\,\mathbf{i} + (2\lambda - 1)\,\mathbf{j}$, where the unit of length is the metre. The bead is moved a distance of $\sqrt{80}$ m along the wire by a force $\mathbf{F} = (4\mathbf{i} - 3\mathbf{j})$ N. Find the magnitude of the work done by $\mathbf{F}$.

(5)

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Question 1 continued

(Total 5 marks)

Q1

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2. Three forces $\mathbf{F}_1 = (a\mathbf{i} + b\mathbf{j} - 2\mathbf{k})$ N, $\mathbf{F}_2 = (-\mathbf{i} + \mathbf{j} - 2\mathbf{k})$ N and $\mathbf{F}_3 = (-\mathbf{i} - 3\mathbf{j} + \mathbf{k})$ N, where a and b are constants, act on a rigid body.

The force $\mathbf{F}_1$ acts through the point with position vector $\mathbf{k}$ m, the force $\mathbf{F}_2$ acts through the point with position vector $(3\mathbf{i} - \mathbf{j} + \mathbf{k})$ m and the force $\mathbf{F}_3$ acts through the point with position vector $(\mathbf{j} + 2\mathbf{k})$ m.

The system of three forces is equivalent to a single force $\mathbf{R}$ acting through the origin together with a couple of moment $\mathbf{G}$. The direction of $\mathbf{R}$ is parallel to the direction of $\mathbf{G}$.

Find the value of a and the value of b.

(11)

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Question 2 continued

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Question 2 continued

Q2

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3. A particle P moves in the xy-plane in such a way that its position vector $\mathbf{r}$ metres at time t seconds, where $0 \leqslant t < \pi$, satisfies the differential equation

$$\sec^2\left(\frac{1}{2}t\right)\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t} + \sec^3\left(\frac{1}{2}t\right)\sin\left(\frac{1}{2}t\right)\mathbf{r} = \sin\left(\frac{1}{2}t\right)\mathbf{i} + \sec^2\left(\frac{1}{2}t\right)\mathbf{j}$$

When $t = 0$, the particle is at the point with position vector $(-\mathbf{i} + \mathbf{j})$ m.

Find $\mathbf{r}$ in terms of t.

(8)

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Question 3 continued

Q3

(Total 8 marks)

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4. A uniform lamina of mass M kg is modelled as the region which is bounded by the curve with equation $y = x^2$, the positive x-axis and the line with equation $x = 2$. The unit of length on both axes is the metre. Find the moment of inertia of the lamina about the x-axis.

(6)

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Question 4 continued

(Total 6 marks)

Q4

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5. At time $t = 0$ a rocket is launched. The rocket has initial mass M, of which mass λM, $0 < \lambda < 1$, is fuel. The rocket is launched vertically upwards, from rest, from the surface of the Earth. The rocket burns fuel and the burnt fuel is ejected vertically downwards with constant speed U relative to the rocket. At time t, the rocket has mass m and velocity v. Ignoring air resistance and any variation in g,

(a) show, from first principles, that until all the fuel is used,

$$m\frac{\mathrm{d}v}{\mathrm{d}t} + U\frac{\mathrm{d}m}{\mathrm{d}t} = -mg$$

(4)

The rocket accelerates vertically upwards with constant acceleration g.

(b) Show that $m = M\mathrm{e}^{\frac{-2gt}{U}}$

(4)

(c) Find, in terms of M, U and λ, an expression for the kinetic energy of the rocket at the instant when all of the fuel has been used.

(6)

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Question 5 continued

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Question 5 continued

Q5

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6. Three equal uniform rods, each of mass m and length $2a$, form the sides of a rigid equilateral triangular frame ABC. The frame is free to rotate in a vertical plane about a fixed smooth horizontal axis L which passes through A and is perpendicular to the plane of the frame.

(a) Show that the moment of inertia of the frame about L is $6ma^2$.

(4)

The frame is held with AB horizontal and C below AB, and released from rest.

Given that the centre of mass of the frame is two thirds of the way along a median from a vertex,

(b) find the magnitude of the force exerted by the axis on the frame at A at the instant when the frame is released.

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Question 6 continued

Q6

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7. A pendulum consists of a uniform circular disc, of radius a and mass $4m$, whose centre is fixed to the end B of a uniform rod AB. The rod has mass $3m$ and length $4l$, where $2l > a$. The rod lies in the same plane as the disc. The pendulum is free to rotate about a fixed smooth horizontal axis L which passes through A and is perpendicular to the plane of the disc. The moment of inertia of the pendulum about L is $2m(a^2 + 40l^2)$.

(a) Find the approximate period of small oscillations of the pendulum about its position of stable equilibrium.

(5)

The pendulum is held with B vertically above A and is then slightly displaced from rest. In the subsequent motion the midpoint of AB strikes a small peg, which is fixed at the same horizontal level as A, and the pendulum rebounds upwards. Immediately before it strikes the peg, the angular speed of the pendulum is ω.

(b) Show that $\omega^2 = \dfrac{22gl}{(a^2 + 40l^2)}$

(3)

Immediately after it strikes the peg, the angular speed of the pendulum is $\frac{1}{2}\omega$.

(c) Find, in terms of m, g, a and l, the magnitude of the impulse exerted on the peg by the pendulum.

(4)

(d) Show that the size of the angle turned through by the pendulum, between it hitting the peg and it next coming to rest, is $\arcsin\frac{1}{4}$.

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Question 7 continued

Q7

(Total 16 marks)

TOTAL FOR PAPER: 75 MARKS

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