GCE MARKING SCHEME

SUMMER 2017

MATHEMATICS - FP1
0977-01

INTRODUCTION

This marking scheme was used by WJEC for the 2017 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conference, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about this marking scheme.

FP1 – June 2017 – Markscheme

Ques	Solution	Mark	Notes
1(a)	det(**M**) = 6 – 4 + 2(3 – 4) +3(8 – 9)	M1	
	= – 3	A1	
(b)(i)	adj(**M**) = $\begin{bmatrix} 2 & 8 & -7 \\ -1 & -7 & 5 \\ -1 & 2 & -1 \end{bmatrix}$	M1A1	Award M1 if at least 5 correct elements
(ii)	$\mathbf{M}^{-1} = -\frac{1}{3}\begin{bmatrix} 2 & 8 & -7 \\ -1 & -7 & 5 \\ -1 & 2 & -1 \end{bmatrix}$	B1	FT if at least one M1 awarded
(c)	$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{3}\begin{bmatrix} 2 & 8 & -7 \\ -1 & -7 & 5 \\ -1 & 2 & -1 \end{bmatrix}\begin{bmatrix} 11 \\ 11 \\ 17 \end{bmatrix}$	M1	FT inverse in (b)(ii)
	$= \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$	A1	
2	$S_n = \sum_{r=1}^{n}(3r-2)^2$	M1	
	$S_n = 9\sum_{r=1}^{n} r^2 - 12\sum_{r=1}^{n} r + 4\sum_{r=1}^{n} 1$	A1	
	$= \frac{9n(n+1)(2n+1)}{6} - \frac{12n(n+1)}{2} + 4n$	A1	
	$= \frac{n\left(9(n+1)(2n+1) - 36(n+1) + 24\right)}{6}$	A1	
	$= \frac{n\left(18n^2 + 27n + 9 - 36n - 36 + 24\right)}{6}$	A1	
	$= 3n^3 - \frac{3}{2}n^2 - \frac{1}{2}n$	A1	
3	EITHER		
	$\|1+2\mathrm{i}\| = \sqrt{5}$; $\|-3+\mathrm{i}\| = \sqrt{10}$; $\|1+3\mathrm{i}\| = \sqrt{10}$	B2	For both moduli and arguments, B1 for 2 correct values
	$\arg(1+2\mathrm{i}) = 1.107; \arg(-3+\mathrm{i}) = 2.820;$ $\arg(1+3\mathrm{i}) = 1.249$	B2	Accept 63.43°, 161.56°,71.56°
	$\|z\| = \frac{\sqrt{5}\times\sqrt{10}}{\sqrt{10}} = \sqrt{5}$ cao	M1A1	
	$\arg(z) = 1.107 + 2.820 - 1.249 = 2.68$ cao	M1A1	Accept 153°

Ques	Solution	Mark	Notes
	OR		
	$\frac{(1+2i)(-3+i)}{(1+3i)} = \frac{(-5-5i)}{(1+3i)}$	(M1A1)	
	$= \frac{(-5-5i)(1-3i)}{(1+3i)(1-3i)}$	(M1)	
	$= \frac{(-20+10i)}{10}$	(A1)	
		(A1)	
	$= -2 + i$	(A1)	
	$\lvert z \rvert = \sqrt{5}$; $\arg(z) = 153^\circ$ or 2.68 rad	(B1B1)	FT from line above provided both M marks awarded and arg is not in the 1st quadrant
4(a)	Reflection matrix = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	**B1**	
	Translation matrix = $\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$	**B1**	
	Rotation matrix = $\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	**B1**	
	$\mathbf{T} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$	**M1**	
	$= \begin{bmatrix} 0 & -1 & -1 \\ 1 & 0 & -2 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ or $\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & -2 \\ 0 & -1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$	**A1**	
	$= \begin{bmatrix} 0 & 1 & -1 \\ 1 & 0 & -2 \\ 0 & 0 & 1 \end{bmatrix}$		Convincing, answer given
(b)	Fixed points satisfy		
	$\begin{bmatrix} 0 & 1 & -1 \\ 1 & 0 & -2 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$	**M1**	
	$x = y - 1$ $y = x - 2$	**A1**	A1 both equations
	These equations have no solution because, for example, $x = y - 1 = y + 2$ therefore no fixed points or algebra leading to 0 = 3 or equivalent	**A1**	Convincing FT from line above provided it leads to no fixed point

Ques	Solution	Mark	Notes
5(a)	Using row operations, $x + 3y - z = 1$ $7y - 4z = -1$ $14y - 8z = 3 - \lambda$ It follows that $3 - \lambda = -2$ $\lambda = 5$	**M1** **A1** **A1** **A1**	
(b)	Let $z = \alpha$ $y = \dfrac{4\alpha - 1}{7}$ $x = \dfrac{10 - 5\alpha}{7}$	**M1** **A1** **A1**	FT from (a)
6	Putting n = 1 states that 8 is divisible by 8 which is correct so true for n = 1. Let the result be true for $n = k$, ie $9^k - 1$ is divisible by 8 or $9^k = 8N + 1$ Consider (for $n = k + 1$) $9^{k+1} - 1 = 9 \times 9^k - 1$ $= 9(8N + 1) - 1$ $= 72N + 8$ Both terms are divisible by 8 Hence true for $n = k \Rightarrow$ true for $n = k + 1$ and since true for n = 1, the result is proved by induction.	**B1** **M1** **M1** **A1** **A1** **A1** **A1**	Only award if all previous marks awarded
7(a)	Taking logs, $\ln f(x) = \tan x \ln \tan x$ Differentiating, $\dfrac{f'(x)}{f(x)} = \sec^2 x \ln \tan x + \dfrac{\tan x \sec^2 x}{\tan x}$ $f'(x) = (\tan x)^{\tan x} \sec^2 x(1 + \ln \tan x)$	**M1** **A1A1** **A1**	A1 for LHS, A1 for RHS
(b)	Stationary points satisfy $1 + \ln\tan x = 0$ $\tan x = \dfrac{1}{e}$ $x = 0.35$	**M1** **A1** **A1**	

Ques	Solution	Mark	Notes
8(a)	$x+\mathrm{i}y=\frac{1}{u+\mathrm{i}v}\times\frac{u-\mathrm{i}v}{u-\mathrm{i}v}$	M1	
	$=\frac{u-iv}{u^2+v^2}$	A1	
	$x=\frac{u}{u^2+v^2};y=\frac{-v}{u^2+v^2}$	A1A1	
(b)(i)	Putting $x + y = 1$ gives	M1	FT from (a)
	$\frac{u-v}{u^2+v^2}=1$	A1	
	$u^2+v^2-u+v=0$	A1	
	This is the equation of a circle		
(ii)	Completing the square,		
	$\left(u-\frac{1}{2}\right)^2+\left(v+\frac{1}{2}\right)^2=\frac{1}{2}$	M1	
	The centre is $\left(\frac{1}{2},-\frac{1}{2}\right)$	A1	
	The radius is $\frac{1}{\sqrt{2}}$	A1	
(c)	Putting $w = z$,	M1	Allow working in terms of x,y,u,v
	$z^2=1$ giving $z=\pm1$	m1	
	The two possible positions are (1,0) and (– 1,0)	A1	

Ques	Solution	Mark	Notes
9(a)(i)	$\alpha+\beta+\gamma=-2$ $\beta\gamma+\gamma\alpha+\alpha\beta=3$ $\alpha\beta\gamma=-4$	**B1**	
	$\frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{1}{\gamma^2}=\frac{\beta^2\gamma^2+\gamma^2\alpha^2+\alpha^2\beta^2}{\alpha^2\beta^2\gamma^2}$	**M1**	
	$=\frac{(\beta\gamma+\gamma\alpha+\alpha\beta)^2-2\alpha\beta\gamma(\alpha+\beta+\gamma)}{\alpha^2\beta^2\gamma^2}$	**A1**	
	$=\frac{3^2-2\times(-4)\times(-2)}{(-4)^2}$	**A1**	
	$=-\frac{7}{16}$		
(ii)	There are two complex roots and one real root	**B1**	Allow a less specific correct comment, eg not all the roots are real
(b)	Let the roots be a,b,c. $a+b+c=\frac{\alpha}{\beta\gamma}+\frac{\beta}{\gamma\alpha}+\frac{\gamma}{\alpha\beta}$		
	$=\frac{\alpha^2+\beta^2+\gamma^2}{\alpha\beta\gamma}$	**M1**	
	$=\frac{(\alpha+\beta+\gamma)^2-2(\beta\gamma+\gamma\alpha+\alpha\beta)}{\alpha\beta\gamma}$	**A1**	
	$=\frac{(-2)^2-2\times3}{(-4)}$		
	$=\frac{1}{2}$	**A1**	
	$bc+ca+ab=\frac{1}{\alpha^2}+\frac{1}{\beta^2}+\frac{1}{\gamma^2}=-\frac{7}{16}$	**B1**	Can be implied by final answer
	$abc=\frac{1}{\alpha\beta\gamma}=-\frac{1}{4}$	**B1**	
	The required equation is $x^3-\frac{1}{2}x^2-\frac{7}{16}x+\frac{1}{4}=0$ (or equivalent)	**M1A1**	FT their previous values Award M1 for correct numbers irrespective of signs