GCE MARKING SCHEME

SUMMER 2017

MATHEMATICS - M3
0982-01

INTRODUCTION

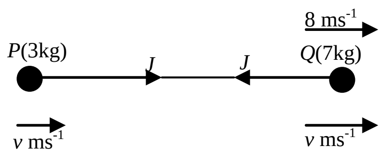
This marking scheme was used by WJEC for the 2017 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conference, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about this marking scheme.

Mathematics M3 (June2017)
Markscheme

Q	Solution	Mark	Notes
1(a)	$\frac{dx}{dt} = 2 - x$		
	$\int \frac{dx}{2-x} = \int dt$	M1	sep variables, (2-x) required
	$-\ln\lvert 2-x\rvert = t + (C)$	A1	correct integration ft x-2
	When $t = 0$, $x = 0$	m1	use of initial conditions
	C = -ln 2	A1	ft if ln present.
	$t = \ln\left\lvert\frac{2}{2-x}\right\rvert$		
	When $x = 1$		
	$t = \ln 2 = (0.693)$	A1	cao
	$e^{-t} = \frac{2-x}{2}$	m1	correct method inversion
	$x = 2(1 - e^{-t})$	A1	any correct exp. cao
1(b)	$\frac{d^2x}{dt^2} = -\frac{dx}{dt}$	M1	
	$\frac{d^2x}{dt^2} = -(2-x) = x - 2$		
	$\frac{d^2x}{dt^2} = 2(1 - e^{-t}) - 2$	m1	substitute for x
	$\frac{d^2x}{dt^2} = -2e^{-t}$	A1	
	Alternative		
	$x = 2(1 - e^{-t})$		
	$\frac{dx}{dt} = 2e^{-t}$	(M1)(A1)	ft similar expressions
	$\frac{d^2x}{dt^2} = -2e^{-t}$	(A1)	ft $\frac{dx}{dt} = -2e^{-t}$ only.

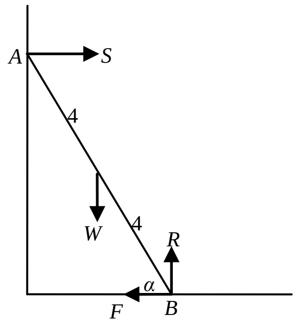
Q	Solution	Mark	Notes
2			
	Impulse = change in momentum Applied to *Q* $J = 7 \times 8 - 7v$	 M1 A1	 allow +/-*J*
	Applied to *P* $J = 3v$	 B1	
	$3v = 56 - 7v$ $v = \underline{5.6\ (\text{ms}^{-1})}$	m1 A1	 cao
	$J = \underline{16.8\ (\text{Ns})}$	A1	cao

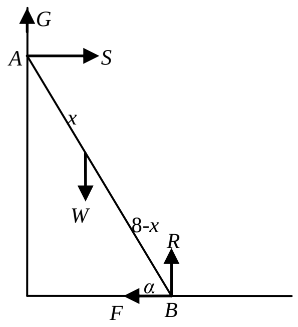
Q	Solution	Mark	Notes
3(a)	$\frac{d^2x}{dt^2} - 6\frac{dx}{dt} + 5x = 0$		
	Auxilliary equation $m^2 - 6m + 5 = 0$	M1	
	$(m - 1)(m - 5) = 0$, $m = 1, 5$		
	G.S. is $x = Ae^t + Be^{5t}$	A1	ft 2 real roots
	When $t = 0$, $x = 8$ and $\frac{dx}{dt} = 16$	m1	used both
	$A + B = 8$		
	$\frac{dx}{dt} = Ae^t + 5Be^{5t}$	B1	ft similar expressions
	$A + 5B = 16$		
	Solving, $A = 6$, $B = 2$	A1	both values cao
	$x = 6e^t + 2e^{5t}$		
3(b)	$\frac{d^2x}{dt^2} - 6\frac{dx}{dt} + 10x = 0$		
	Auxilliary equation $m^2 - 6m + 10 = 0$	M1	
	$m = 3 \pm i$		
	C.F. is $x = e^{3t}(A\sin t + B\cos t)$	A1	ft complex roots
	Using initial conditions	m1	used both
	$B = 8$		
	$\frac{dx}{dt} = 3e^{3t}(A\sin t + B\cos t) + e^{3t}(A\cos t - B\sin t)$	B1	ft similar expression
	$16 = 24 + A$, $A = -8$	A1	both values cao
	$x = 8e^{3t}(-\sin t + \cos t)$		
3(c)	$\frac{d^2x}{dt^2} - 6\frac{dx}{dt} = (12t - 26)$,		
	Auxilliary equation $m^2 - 6m = 0$	M1	
	$m = 0, 6$		
	C.F. is $x = A + Be^{6t}$	A1	ft 0, another real root
	For P.I. try $x = at^2 + bt$	M1	allow $at+b$
	$2a - 6(2at + b) = 12t - 26$	A1	correct LHS
	$a = -1$	m1	comparing coefficients
	$2a - 6b = -26$, $b = 4$	A1	both values cao
	$x = A + Be^{6t} - t^2 + 4t$		
	$8 = A + B$		
	$\frac{dx}{dt} = 6Be^{6t} - 2t + 4$	B1	ft similar CF+PI
	$16 = 6B + 4$		
	$B = 2$, $A = 6$	A1	both values cao
	$x = 2e^{6t} - t^2 + 4t + 6$		

Q	Solution	Mark	Notes
4(a)	N2L applied to P	M1	Dimensionally correct All forces
	$-3v^2 = 0.5\ \frac{\mathrm{d}v}{\mathrm{d}t}$		
	$\frac{\mathrm{d}v}{\mathrm{d}t} = -6v^2$	A1	convincing
4(b)	$-\int \frac{dv}{v^2} = 6\int dt$	M1	separating variables
	$\frac{1}{v} = 6t + (C)$	A1	correct integration
	When t=0, v=2	m1	use of initial conditions
	$C = \frac{1}{2}$		
	$\frac{1}{v} = 6t + \frac{1}{2}$		
	$v = \frac{2}{12t+1}$	A1	cao, any correct exp.
4(c)	$v\frac{\mathrm{d}v}{\mathrm{d}x} = -6v^2$	M1	
	$\frac{\mathrm{d}v}{\mathrm{d}x} = -6v$		
	$\int \frac{dv}{v} = -6\int dx$	m1	separating variables
	$\ln v = -6x + (C)$	A1	correct integration
	when $x = 0$, $v = 2$	m1	use of initial conditions
	$C = \ln 2$		
	$-6x = \ln v - \ln 2$		
	$v = 2e^{-6x}$	A1	cao, any correct exp.
4(d)	Rate of work = $F.v$	M1	used
	Rate of work = $3v^2 \times v$	A1	
	Rate of work = $3(2e^{-6x})^3$		
	Rate of work = $24e^{-18x}$	A1	cao, any correct exp.

Q	Solution	Mark	Notes
5(a)	$v^2 = -4x^2 + 8x + 21$	M1	attempt to differentiate
	$2v\dfrac{dv}{dx} = -8x + 8$	A1	or dv/dx=
	$v\dfrac{dv}{dx} = -4(x-1)$		
	$\dfrac{d^2x}{dt^2} = -4(x-1)$	A1	
	Let $y = x - 1$, $\dfrac{dy}{dt} = \dfrac{dx}{dt}$, $\dfrac{d^2y}{dt^2} = \dfrac{d^2x}{dt^2}$,		
	$\dfrac{d^2y}{dt^2} = -4y = -2^2y$		
	Hence motion is simple harmonic	A1	convincing
	Centre of motion is $x = 1$	B1	
5(b)	$\omega = 2$	B1	
	Period = $\dfrac{2\pi}{2} = \pi$	B1	convincing
	Amplitude is given by x-1 when $v = 0$ $-4x^2 + 8x + 21 = -4(x-1)^2 + 25 = 0$ $(x-1) = \pm 2.5$	M1	v=0
	Amplitude = a = 2.5	A1	cao
	<u>Alternative solution</u> $v^2 = \omega^2[a^2 - y^2]$	(M1)	attempt to write equation in correct form
	$v^2 = 2^2[2.5^2 - (x-1)^2]$		
	Hence $\omega = 2$	(B1)	
	Period = $\dfrac{2\pi}{2} = \pi$	(B1)	
	Amplitude = a = 2.5	(A1)	cao
	<u>Alternative solution</u> Amplitude is given when $v = 0$	(M1)	used
	$-4x^2 + 8x + 21 = 0$ $(2x + 3)(2x - 7) = 0$ $x = -1.5, 3.5$		
	amplitude = 3.5 – 1 = 2.5	(A1)	cao

5(c)	$(x - 1) = 2.5 \sin(2t)$	M1	
	$x = 2.5 \sin(2t) + 1$		
	$3 - 1 = 2.5 \sin(2t)$	m1	use of 3-centre
	$2t = \sin^{-1}\left(\frac{2}{2.5}\right)$	m1	inversion ft a,ω,centre
	$2t = 0.927295$		
	$t = \underline{0.4636\ (s)}$	A1	cao

Q	Solution	Mark	Notes
6(a)			
	Resolve vertically		
	$R = W$	B1	
	Resolve horizontally		
	$S = F = \mu R = \mu W$	B1	
	Moments about B		
	$W\times 4\cos\alpha = S\times 8\sin\alpha$	M1	dim correct, all forces no extra except friction A
	$16W = \mu W\times 8\times 3$		
	$\mu = \frac{2}{3}$	A1	cao

Q	Solution	Mark	Notes
6(b)	 		
	$F = 0.6R$ $G = 0.6S$	B1	both
	Resolve vertically	M1	dimensionally correct All forces, no extra
	$G + R = W$ $0.6S + R = W$	A1	
	Resolve horizontally	M1	dimensionally correct All forces, no extra
	$S = F$ $S = 0.6R$	A1	
	$0.6 \times 0.6R + R = W$ $1.36R = W$		
	Moments about A	M1	dimensionally correct All forces, no extra
	$Wx\cos\alpha + 0.6R \times 8\sin\alpha = R \times 8\cos\alpha$	A2	-1 each error
	$1.36Rx\frac{4}{5} + 4.8R \times \frac{3}{5} = 8R \times \frac{4}{5}$	m1	substitute to obtain one common factor force
	$5.44x + 14.4 = 32$ $5.44x = 17.6$		
	$x = \frac{55}{17} = \underline{3.2353 \text{ (m)}}$	A1	cao