wjec cbac

GCE AS/A Level

0983/01

S17-0983-01

MATHEMATICS – S1
Statistics

WEDNESDAY, 14 JUNE 2017 – MORNING

1 hour 30 minutes

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:
- a WJEC pink 16-page answer booklet;
- a Formula Booklet;
- a calculator;
- statistical tables (Murdoch and Barnes or RND/WJEC Publications).

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. The events A and B are such that

$$P(A) = 0{\cdot}2,\ P(B) = 0{\cdot}3,\ P(A \cup B) = 0{\cdot}4.$$

(a) Show that A and B are not independent. [3]

(b) Determine the value of

(i) $P(A'|B)$,

(ii) $P(A \cup B')$. [6]

2. The random variable X has mean 10 and standard deviation 2.

(a) Find the value of $E(X^2)$. [2]

(b) If $Y = 2X + 3$, find the mean and the variance of Y. [4]

3. A box contains nine cards of which four are white, three are red and two are blue. Three of these cards are selected at random without replacement.
Calculate the probability that

(a) one card of each colour is selected, [3]

(b) the three cards selected are all of the same colour. [3]

4. Independently for each page, the number of typing errors per page in the first draft of a book has a Poisson distribution with mean 0·8.

(a) (i) Calculate the probability that a randomly chosen page contains at least one error.

(ii) Calculate the probability that the third page of three randomly chosen pages is the first to contain at least one error. [4]

(b) (i) Write down the probability p_n that a random selection of n pages contains no errors.

(ii) Find the smallest value of n such that $p_n < 0{\cdot}001$. [6]

5. Anne and Brian play a board game against each other regularly.

(a) The probability that Anne wins a game is 0·7 and the probability that Brian wins a game is 0·3, independently of all other games. One day, they play 10 games. Let X denote the number of games won by Anne on that day.

(i) State the distribution of X, including any parameters.

(ii) Determine the mean and the standard deviation of X.

(iii) Find the probability that Anne wins more games than Brian. [7]

(b) The probability that one of their games takes more than 1 hour to complete is 0·06. During a school holiday, they play 44 games. Use a Poisson approximation to find the probability that more than 2 of these games take more than 1 hour to complete. [3]

6. The discrete random variable X has the following probability distribution.

$$P(X = x) = \frac{x^2}{54} \qquad \text{for } x = 2, 3, 4, 5,$$
$$P(X = x) = 0 \qquad \text{otherwise.}$$

(a) Calculate the mean and the variance of X. [5]

(b) Three independent observations X_1, X_2, X_3 are taken from the distribution of X. Determine the value of $P(X_1 + X_2 + X_3 = 14)$. [4]

7. It is known that 5% of animals of a certain species have a particular disease. A diagnostic test can be applied to animals of this species to indicate whether or not they have this disease. When applied to an animal which has this disease, the test gives a positive response with probability 0·96. When applied to an animal which does not have this disease, the test gives a positive response with probability 0·02.

(a) The test is given to a randomly chosen animal.

(i) Calculate the probability that a positive response is obtained. [3]

(ii) Given that a positive response is obtained, find the probability that this animal has the disease. [3]

(b) A randomly chosen animal gave a positive response when tested. It is tested again.

(i) Find the probability that it gives a second positive response.

(ii) Given that this second response is positive, calculate the probability that this animal has the disease. [4]

8. The continuous random variable X has cumulative distribution function F given by

$$F(x) = 0 \qquad \text{for } x < 1,$$
$$F(x) = k(x^4 - x^2) \qquad \text{for } 1 \leqslant x \leqslant 2,$$
$$F(x) = 1 \qquad \text{for } x > 2,$$

where k is a constant.

(a) (i) Show that $k = \frac{1}{12}$.

(ii) Find the 95th percentile of X, giving your answer correct to three significant figures.

(iii) Evaluate $P(X < 1{\cdot}25 | X < 1{\cdot}75)$. [9]

(b) (i) Find an expression for $f(x)$, valid for $1 \leqslant x \leqslant 2$, where f denotes the probability density function of X.

(ii) Calculate $E\left(\sqrt{X}\right)$. [6]

END OF PAPER