GCE MARKING SCHEME

SUMMER 2016

Mathematics – FP2
0978/01

INTRODUCTION

This marking scheme was used by WJEC for the Summer 2016 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

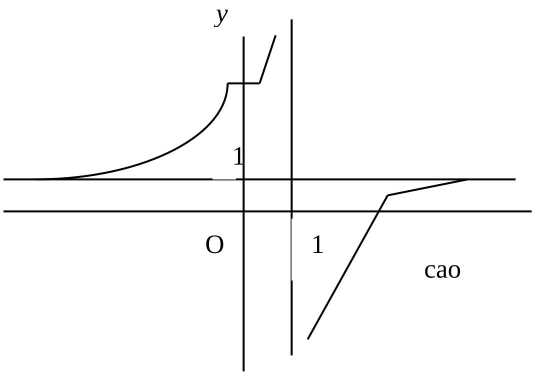
It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conference, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about this marking scheme.

Ques	Solution	Mark	Notes
1	Putting $u = x^2$, $du = 2x dx$, $[0, \sqrt{2}]$ becomes $[0,2]$ $I = \frac{1}{2}\int_0^2 \frac{du}{\sqrt{(16-u^2)}}$ $= \frac{1}{2}\left[\sin^{-1}\left(\frac{u}{4}\right)\right]_0^2$ $= \frac{1}{2}\sin^{-1}\left(\frac{1}{2}\right)$ $= \frac{\pi}{12}$	 **B1B1** **M1** **A1** **A1** **A1**	 Valid attempt to substitute
2(a)(i) (ii) (b)	$(3-i)^2 = 9-6i-1 = 8-6i$ $(3-i)^4 = (8-6i)^2 = 64-96i-36 = 28-96i$ The 4[th] roots are 3 – i and –3 + i and 1 + 3i, – 1 –3i	**M1A1** **B1** **B1** **B1B1**	 Convincing Must start with 3 – i and rotate
3(a) (b)	$\cos 4\theta + i\sin 4\theta = (\cos\theta + i\sin\theta)^4$ $= 4i\cos^3\theta\sin\theta - 4i\cos\theta\sin^3\theta$ + real terms $\sin 4\theta = 4\cos^3\theta\sin\theta - 4\cos\theta\sin^3\theta$ $\frac{\sin 4\theta}{\sin\theta} = 4\cos\theta(1-\sin^2\theta-\sin^2\theta)$ $= 4\cos\theta(1-2\sin^2\theta)$ EITHER $\int_{\pi/6}^{\pi/4}\frac{\sin 4\theta}{\sin\theta}d\theta = 4\int_{\pi/6}^{\pi/4}\cos\theta\cos 2\theta d\theta$ $= 2\int_{\pi/6}^{\pi/4}[\cos\theta + \cos 3\theta]d\theta$ $= 2\left[\sin\theta + \frac{\sin 3\theta}{3}\right]_{\pi/6}^{\pi/4}$ = 0.219 OR $\int_{\pi/6}^{\pi/4}\frac{\sin 4\theta}{\sin\theta}d\theta = 4\int_{\pi/6}^{\pi/4}(1-2\sin^2\theta)d\sin\theta$ $= 4\left[\sin\theta - \frac{2}{3}\sin^3\theta\right]_{\pi/6}^{\pi/4}$ = 0.219	**M1** **m1** **A1** **A1** **M1** **A1** **A1** **A1** **(M1A1)** **(A1)** **(A1)**	 This line must be seen This line must be seen

Ques	Solution	Mark	Notes
4	Substituting $t = \tan\left(\frac{x}{2}\right)$,		
	$\frac{2t}{1+t^2} + \frac{2t}{1-t^2} + t = 0$	M1A1	
	$\frac{2t(1-t^2) + 2t(1+t^2) + t(1+t^2)(1-t^2)}{(1+t^2)(1-t^2)} = 0$	A1	
	$\frac{2t - 2t^3 + 2t + 2t^3 + t - t^5}{(1+t^2)(1-t^2)} = 0$	A1	
	$t(5 - t^4) = 0$	A1	
	$t = 0$	B1	FT for $t^4 = n$
	$\frac{x}{2} = 0 + n\pi$ giving $x = 2n\pi$	B1	Penalise – 1for use of degrees throughout
	$t = \sqrt[4]{5}$	B1	
	$\frac{x}{2} = 0.981 + n\pi$ giving $x = 1.96 + 2n\pi$	B1	
	$t = -\sqrt[4]{5}$	B1	
	$\frac{x}{2} = -0.981 + n\pi$ giving $x = -1.96 + 2n\pi$	B1	
5(a)	Because $f(-x)$ is neither equal to $f(x)$ or $-f(x)$, f is neither even nor odd.	B1	
(b)	Let		
	$\frac{3x^2 + x + 6}{(x+2)(x^2+4)} = \frac{A}{x+2} + \frac{Bx + C}{x^2+4}$	M1	
	$= \frac{A(x^2+4) + (x+2)(Bx+C)}{(x+2)(x^2+4)}$	A1	
	$A = 2$; $B = 1$; $C = -1$	A1A1A1	
(c)			FT their values from (a)
	$\int_0^1 f(x)\,\mathrm{d}x = \int_0^1 \frac{2}{x+2}\mathrm{d}x + \int_0^1 \frac{x}{x^2+4}\mathrm{d}x - \int_0^1 \frac{1}{x^2+4}\mathrm{d}x$	M1	
	$= 2\left[\ln(x+2)\right]_0^1 + \frac{1}{2}\left[\ln(x^2+4)\right]_0^1 - \frac{1}{2}\left[\tan^{-1}\left(\frac{x}{2}\right)\right]_0^1$	A1A1A1	
	$= 2\ln 3 - 2\ln 2 + \frac{1}{2}\ln 5 - \frac{1}{2}\ln 4 - \frac{1}{2}\tan^{-1}\left(\frac{1}{2}\right)$	A1	
	$= 0.691$	A1	

Ques	Solution	Mark	Notes
6(a)	If $x = a\sec\theta$ and $y = b\tan\theta$, then $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = \sec^2\theta - \tan^2\theta = 1$ showing that the point $(a\sec\theta, b\tan\theta)$ lies on the hyperbola.	M1A1	
(b)(i)	EITHER $\dfrac{dx}{d\theta} = \sec\theta\tan\theta, \dfrac{dy}{d\theta} = \sec^2\theta$	M1	
	$\dfrac{dy}{dx} = \dfrac{\sec^2\theta}{\sec\theta\tan\theta}$	A1	
	$= \operatorname{cosec}\theta$	A1	
	OR $2x - 2y\dfrac{dy}{dx} = 0$	(M1)	
	$\dfrac{dy}{dx} = \dfrac{x}{y}$	(A1)	
	$= \dfrac{\sec\theta}{\tan\theta} = \operatorname{cosec}\theta$	(A1)	
	The gradient of the normal is $-\sin\theta$.	M1	
	The equation of the normal is $y - \tan\theta = -\sin\theta(x - \sec\theta)$ $x\sin\theta + y = 2\tan\theta$	A1	
(ii)	The normal meets the x-axis where $y = 0$, ie $x = 2\sec\theta, y = 0$	B1	
	The coordinates of the midpoint of PQ are $\left(\dfrac{\sec\theta + 2\sec\theta}{2}, \dfrac{\tan\theta + 0}{2}\right)$, ie	M1	
	$\left(\dfrac{3}{2}\sec\theta, \dfrac{1}{2}\tan\theta\right)$ cao	A1	
	EITHER This is the parametric form of a hyperbola showing that the locus of the midpoint is a hyperbola	A1	FT from midpoint
	OR $x = \dfrac{3}{2}\sec\theta, y = \dfrac{1}{2}\tan\theta$		FT from midpoint
	$\Rightarrow \sec\theta = \dfrac{2}{3}x, \tan\theta = 2y$		
	$\Rightarrow \dfrac{x^2}{9/4} - \dfrac{y^2}{1/4} = 1$		
	This is the equation of a hyperbola showing that the locus of the midpoint is a hyperbola	(A1)	
	Since $a = 3/2$ and $b = ½$, Eccentricity = $\sqrt{\dfrac{1.5^2 + 0.5^2}{1.5^2}} = \dfrac{\sqrt{10}}{3}$	M1A1	
	The coordinates of the foci are $\left(\pm\dfrac{\sqrt{10}}{2}, 0\right)$	A1	

Ques	Solution	Mark	Notes
7(a)	$x = 1$ cao $y = 1$ cao	B1 B1	Penalise – 1for extra asymptotes
(b)	$f(0) = 8$ giving the point (0,8) cao $f(x) = 0 \Rightarrow x = 2$ giving the point (2,0) cao	B1 B1	
(c)	$f'(x) = \dfrac{3x^2(x^3-1)-3x^2(x^3-8)}{(x^3-1)^2}\left(=\dfrac{21x^2}{(x^3-1)^2}\right)$ The stationary point is (0,8). $f'(x) > 0$ on either side of the stationary point. It is a point of inflection.	M1A1 A1 M1 A1	
(d)	 	G1 G1 G1	RH branch approach to asymptotes LH branch approach to asymptotes Stationary point of inflection
(e)(i)	$f(-2) = 16/9$, $f(2) = 0$ $f(S) = (-\infty,0]\cup[16/9,\infty)$ cao	B1 B1	
(ii)	$f(x) = -2 \Rightarrow x = \sqrt[3]{10/3}$ $f(x) = 2 \Rightarrow x = -\sqrt[3]{6}$ $f^{-1}(S) = (-\infty,-\sqrt[3]{6}]\cup[\sqrt[3]{10/3},\infty)$ cao	M1A1 A1 A1	Accept 1.82 for $\sqrt[3]{6}$ and 1.49 for $\sqrt[3]{10/3}$