GCE AS/A level

0979/01

S16-0979-01

MATHEMATICS – FP3
Further Pure Mathematics

A.M. WEDNESDAY, 29 June 2016

1 hour 30 minutes

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. The curve *C* has polar equation

$$r = 1 + 2\tan\theta\ ,\ 0 \leqslant \theta \leqslant \frac{\pi}{4}\ .$$

Show that there is no point on *C* at which the tangent is perpendicular to the initial line. [7]

2. The function f is defined by $f(x) = \cos x + \cosh x$.

(a) Show that $f^{(4)}(x) = f(x)$, where $f^{(4)}(x)$ denotes the fourth derivative of $f(x)$. [2]

(b) (i) Show that the Maclaurin series of $f(x)$ contains only terms of the form x^{4n}, where n is a non-negative integer.

(ii) Determine the first three non-zero terms of this Maclaurin series. [3]

(c) (i) Hence find an approximate value for the positive root of the equation

$$12(\cos x + \cosh x) - x^4 = 36.$$

Give your answer correct to three significant figures.

(ii) Show that this approximation is the value of the root correct to three significant figures. [5]

3. Using the substitution $t = \tan\left(\frac{x}{2}\right)$, evaluate the integral

$$\int_0^{\frac{\pi}{2}} \frac{\mathrm{d}x}{3+5\cos x}\ ,$$

giving your answer in the form $\ln(3^a)$, where a is a rational number to be determined. [8]

4. The function f is defined on the domain $[0, \infty)$ by

$$f(\theta) = \cosh 2\theta - 8\cosh\theta\, .$$

Consider the equation $f(\theta) = k$, where k is a constant.

(a) Show that the equation has no real roots if $k < -9$. [4]

(b) Solve the equation when $k = -8$, giving your answers correct to two decimal places. [3]

(c) Determine

(i) the value of k for which the equation has a repeated root,

(ii) the set of values of k for which the equation has exactly one real root. [5]

5. The curve C has equation $y = \ln(1 + \cos x)$.

(a) Show that

$$1+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 = \frac{2}{1+\cos x}\ .$$ [4]

(b) Find the length of the arc joining the points $(0, \ln 2)$ and $\left(\frac{\pi}{2}, 0\right)$ on C.

Give your answer in the form $\ln\left(a+b\sqrt{2}\right)$, where a, b are positive integers. [6]

6. The equation

$$x^5 + \sinh x = 3$$

has a root α close to 1.

(a) It is suggested that iterative sequences based on the following rearrangements of the equation could be used to find the value of α.

I. $x = \left(3-\sinh x\right)^{\frac{1}{5}}$

II. $x = \sinh^{-1}\left(3-x^5\right)$

(i) By evaluating appropriate derivatives, show that one of these sequences is convergent and the other is divergent.

(ii) Taking $x_0 = 1$, use the convergent sequence to find the value of α correct to three decimal places. [12]

(b) Use the Newton-Raphson method to find the value of α correct to six decimal places. [6]

7. The integral I_n is given, for $n \geqslant 0$, by

$$I_n = \int_0^{\pi} x^n \sin 2x \,\mathrm{d}x.$$

(a) Show that, for $n \geqslant 2$,

$$I_n = -\frac{\pi^n}{2} - \frac{n(n-1)}{4} I_{n-2}\ .$$ [6]

(b) Evaluate I_4, giving your answer correct to the nearest integer. [4]

END OF PAPER