GCE MARKING SCHEME

SUMMER 2016

Mathematics – FP3
0979/01

INTRODUCTION

This marking scheme was used by WJEC for the Summer 2016 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conference, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about this marking scheme.

Ques	Solution	Mark	Notes
1	Consider $x = r\cos\theta$ $= \cos\theta(1+2\tan\theta) = \cos\theta + 2\sin\theta$ $\frac{dx}{d\theta} = -\sin\theta + 2\cos\theta$ (The tangent is perpendicular to the initial line where) $\frac{dx}{d\theta} = 0$. $\sin\theta = 2\cos\theta$ $\tan\theta = 2$ $\theta = 1.11\ (63^\circ)$ This lies outside the domain for the curve, hence no point at which the tangent is perpendicular to the initial line.	**M1** **A1** **B1** **M1** **A1** **A1** **A1**	or $0 \le \theta \le \frac{\pi}{4} \Rightarrow 0 \le \tan\theta \le 1$
2(a)	$f(x) = \cos x + \cosh x$ $f'(x) = -\sin x + \sinh x$ $f''(x) = -\cos x + \cosh x$ $f'''(x) = \sin x + \sinh x$ $f^{(4)}(x) = \cos x + \cosh x (= f(x))$	B1 B1	Convincing
(b)(i)	$f(0) = 2$ $f'(0) = 0$ $f''(0) = 0$ $f'''(0) = 0$ $f^{(4)}(0) = 2$ This pattern repeats itself every four differentiations so $f^{(n)}(0) = 2$ if n is a multiple of 4 and zero otherwise. (Therefore the only terms in the Maclaurin series are those for which the power is a multiple of 4.)	B1 B1	Accept unsimplified expressions
(ii)	The first three terms are $2, \frac{x^4}{12}, \frac{x^8}{20160}$	B1	
(c)(i)	Substituting the series, $24 + x^4 + \frac{x^8}{1680} - x^4 = 36$ $x^8 = 20160$ $x = 3.45$	M1 A1 A1	
(ii)	Let $g(x) = 12(\cos x + \cosh x) - x^4 - 36$ Consider $g(3.445) = -0.0507\ldots$ $g(3.455) = 0.2312\ldots$ The change of sign confirms that the value of the root is 3.45 correct to 3 significant figures.	B1 B1	

Ques	Solution	Mark	Notes
3	Putting $t = \tan\left(\frac{x}{2}\right)$		
	$[0,\pi/2]$ becomes $[0,1]$	B1	
	$dx = \frac{2dt}{1+t^2}$	B1	
	$I = \int_0^1 \frac{2dt/(1+t^2)}{3+5(1-t^2)/(1+t^2)}$	M1A1	
	$= \int_0^1 \frac{2dt}{8-2t^2}$	A1	
	$= \int_0^1 \frac{dt}{4-t^2}$	A1	
	$= \frac{1}{4}\left[\ln\left(\frac{2+t}{2-t}\right)\right]_0^1$	A1	
	$= \frac{1}{4}\ln 3 = \ln 3^{1/4}$	A1	
4(a)	The equation is $\cosh 2\theta - 8\cosh\theta - k = 0$ Substituting for $\cosh 2\theta$,	M1	
	$2\cosh^2\theta - 8\cosh\theta - (k+1) = 0$	A1	
	$\cosh\theta = \frac{8 \pm \sqrt{72+8k}}{4}$	m1	
	If $k < -9$, $72 + 8k < 0$ so no real solutions.	A1	
(b)	If $k = -8$, $\cosh\theta = \frac{8\pm\sqrt{8}}{4} = 1.292\ldots, 2.707\ldots$	M1A1	
	$\theta = 0.75, 1.65$	A1	Allow $\pm$
(c)(i)	There is a repeated root when $k = -9$	B1	
(ii)	There will be only one real root if the smaller root of the quadratic equation in (a) < 1, ie	M1	
	$\frac{8-\sqrt{72+8k}}{4} < 1$	A1	
	$\sqrt{72+8k} > 4$	M1	Allow $k = -9$ to be included here
	$k > -7$	A1	

Ques	Solution	Mark	Notes
5(a)	$\dfrac{dy}{dx} = -\dfrac{\sin x}{1+\cos x}$	**B1**	
	$1+\left(\dfrac{dy}{dx}\right)^2 = 1+\dfrac{\sin^2 x}{(1+\cos x)^2}$	**M1**	
	$= \dfrac{1+2\cos x+\cos^2 x+\sin^2 x}{(1+\cos x)^2}$	**A1**	
	$= \dfrac{2+2\cos x}{(1+\cos x)^2}$	**A1**	
	$= \dfrac{2}{(1+\cos x)}$		
(b)	METHOD 1		
	Arc length $= \sqrt{2}\displaystyle\int_0^{\pi/2}\sqrt{\frac{1}{(1+\cos x)}}\,dx$	**M1**	
	$= \sqrt{2}\displaystyle\int_0^{\pi/2}\sqrt{\frac{1}{2\cos^2(x/2)}}\,dx$	**m1**	
	$= \displaystyle\int_0^{\pi/2}\sec(x/2)\,dx$	**A1**	
	$= 2\big[\ln(\sec(x/2)+\tan(x/2))\big]_0^{\pi/2}$	**A1**	
	$= 2\ln(1+\sqrt{2})$	**A1**	Award this A1 if the 2 is missing
	$= \ln(3+2\sqrt{2})$	**A1**	
	METHOD 2		
	Arc length $= \sqrt{2}\displaystyle\int_0^{\pi/2}\sqrt{\frac{1}{(1+\cos x)}}\,dx$	M1	
	Put $t=\tan\left(\dfrac{x}{2}\right); dx = \dfrac{2dt}{1+t^2}$	m1	
	Arc length $= \sqrt{2}\displaystyle\int_0^{1}\sqrt{\frac{1}{(1+(1-t^2)/(1+t^2))}}\times\frac{2dt}{1+t^2}$	A1	
	$= 2\displaystyle\int_0^{1}\sqrt{\frac{1}{(1+t^2)}}\,dt$	A1	
	$= 2\ln\left[t+\sqrt{1+t^2}\right]_0^1$	A1	Allow $\sinh^{-1}(t)$
	$= 2\ln\left[1+\sqrt{2}\right] = \ln(3+2\sqrt{2})$	A1	

Ques	Solution	Mark	Notes
6(a)(i)	Let $f(x) = (3 - \sinh x)^{\frac{1}{5}}$		
	$f'(x) = \frac{1}{5}(3 - \sinh x)^{-\frac{4}{5}} \times (-\cosh x)$	M1A1	
	$f'(1) = -0.1907...$	A1	
	Since this is less than 1 in modulus, the sequence is convergent.	A1	
	Let $g(x) = \sinh^{-1}(3 - x^5)$		
	$g'(x) = \frac{1}{\sqrt{1 + (3 - x^5)^2}} \times (-5x^4)$	M1A1	
	$g'(1) = -2.236...$	A1	
	Since this is greater than 1 in modulus, the sequence is divergent.	A1	
(ii)	Successive approximations are 1 1.127828325 1.100939212 1.107049937 1.105684578 1.105990816	M1A1	
	(since the sequence oscillates) the value of the root is 1.106 correct to three decimal places.	A1 A1	
(b)	The Newton-Raphson iteration is $x \to x - \frac{x^5 + \sinh x - 3}{5x^4 + \cosh x}$	M1A1	
	Successive approximations are 1 1.126056647 1.106546041 1.105935334 1.105934755 1.105934754	M1A1 A1	Allow any starting value This last value must be seen for A1
	The value of the root is 1.105935 correct to six decimal places.	A1	

Ques	Solution	Mark	Notes
7(a)	$I_n = -\frac{1}{2}\int_0^{\pi} x^n \mathrm{d}(\cos 2x)$	M1	
	$= -\frac{1}{2}\left[x^n \cos 2x\right]_0^{\pi} + \frac{1}{2}\int_0^{\pi} nx^{n-1}\cos 2x\mathrm{d}x$	A1A1	
	$= -\frac{\pi^n}{2} + \frac{n}{4}\int_0^{\pi} x^{n-1}\mathrm{d}(\sin 2x)$	M1	
	$= -\frac{\pi^n}{2} + \frac{n}{4}\left[x^{n-1}\sin 2x\right]_0^{\pi} - \frac{n(n-1)}{4}I_{n-2}$	A1A1	
	$= -\frac{\pi^n}{2} - \frac{n(n-1)}{4}I_{n-2}$		
(b)	$I_0 = \int_0^{\pi}\sin 2x\mathrm{d}x = -\frac{1}{2}\left[\cos 2x\right]_0^{\pi} = 0$	B1	
	$I_4 = -\frac{\pi^4}{2} - 3I_2$	M1	
	$= -\frac{\pi^4}{2} - 3\left(-\frac{\pi^2}{2} - \frac{1}{2}I_0\right)$	A1	FT their I_0 for this A1
	$= -34$ cao	A1	