



GCE AS/A level

0982/01



S15-0982-01

MATHEMATICS – M3
Mechanics

A.M. MONDAY, 22 June 2015

1 hour 30 minutes

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Take g as 9.8 ms^{-2} .

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. A particle of mass 400 kg moves along a straight horizontal road under the action of a horizontal force F . The magnitude of the force F may be modelled by $500\left(\frac{x}{v+2}\right)$ N, where $v \text{ ms}^{-1}$ is the speed of the particle and $x \text{ m}$ is the distance of the particle from a point O on the road.

(a) Show that the motion of the particle satisfies the differential equation

$$4v(v+2)\frac{dv}{dx} = 5x. \quad [2]$$

(b) When $x = 0$, the particle is at rest.

(i) Find an expression for x in terms of v .

(ii) Find the distance of the particle from O and the acceleration of the particle when its speed is 3 ms^{-1} . [9]

2. (a) An object of mass 0.5 kg is initially moving along the positive x -axis away from the origin O . The object moves under the action of a force of magnitude $6.5x \text{ N}$ which is directed towards O . The resistance to motion of the object is $2v \text{ N}$, where $v \text{ ms}^{-1}$ is the velocity of the object at time t seconds.

(i) Show that the equation of motion of the object is

$$\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 13x = 0.$$

(ii) Find an expression for x in terms of t given that $x = 6$ and $\frac{dx}{dt} = 3$ when $t = 0$.

Determine the approximate value of x when t is large. [9]

(b) Find the general solution of the differential equation

$$\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 13x = 91t + 15. \quad [4]$$

3. A body of mass 250 kg is dropped from a hot air balloon and falls vertically downwards. During the downward motion, the body is subjected to a resistance to motion of $50v \text{ N}$, where $v \text{ ms}^{-1}$ is the speed of the body at time t seconds. The initial speed of the body may be assumed to be zero.

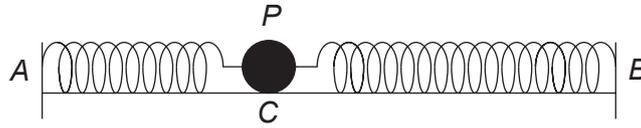
(a) Show that the motion of the body satisfies the differential equation

$$5\frac{dv}{dt} = 5g - v. \quad [2]$$

(b) Find an expression for v in terms of t . Determine the speed of the body when $t = 5$. [7]

(c) Find an expression for x , the distance in metres fallen by the body in t seconds. Hence calculate the distance fallen by the body in 5 seconds. [5]

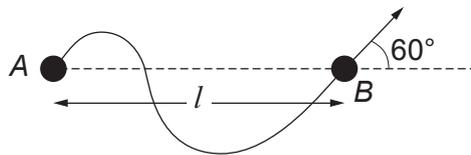
4. The diagram shows a particle P , of mass 7.5 kg , lying on a smooth horizontal surface. It is attached by two light springs to points A and B where AB is 1.4 m . Spring AP has natural length 0.3 m and modulus of elasticity 15 N . Spring BP has natural length 0.6 m and modulus of elasticity 20 N .



When P is in equilibrium, it is at the point C .

- (a) Show that $AC = 0.5 \text{ m}$. [5]
- (b) The particle P is pulled horizontally towards B a distance 0.25 m from C and released.
- Show that the subsequent motion of the particle is Simple Harmonic with period $\frac{3\pi}{5}$ seconds.
 - Write down the amplitude of the motion.
 - Determine the speed of P when it is 0.2 m from C .
 - Find the shortest time taken for P to reach a position where it is 0.2 m from C . [12]

5. Two particles A and B , of mass 3 kg and 5 kg respectively, are attached one to each end of a light inextensible string of length $\sqrt{3}l \text{ m}$. Initially, the particles are at rest on a smooth horizontal surface a distance $l \text{ m}$ apart, as shown in the diagram. Particle B is then projected horizontally with speed 8 ms^{-1} at an angle of 60° to the line joining the initial positions of A and B produced.



Immediately after the string becomes taut,

- show that the particle A starts to move in a direction which makes an angle of 30° with the line joining the initial positions of A and B . [2]
- find the speed with which each particle begins to move and determine the magnitude of the impulsive tension in the string. [9]

TURN OVER

6. A uniform ladder of mass 20 kg and length 6 m rests with its top end against a smooth vertical wall and its bottom end on rough horizontal ground. The ladder is inclined at an angle θ to the horizontal. The coefficient of friction between the ladder and the ground is 0.6. A man of mass 80 kg climbs the ladder. When he reaches $\frac{5}{6}$ of the way up, the ladder is in limiting equilibrium.

Calculate the normal reaction at the wall and the value of θ . State one modelling assumption you have made about the ladder in your solution. [9]

END OF PAPER