



GCE AS/A level

0978/01

MATHEMATICS – FP2
Further Pure Mathematics

P.M. WEDNESDAY, 18 June 2014

1 hour 30 minutes

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. The function f is defined by

$$f(x) = \frac{x^2 + 1}{x(x^2 + 2)}.$$

- (a) Determine whether f is even, odd or neither even nor odd. [3]
 (b) Express $f(x)$ in partial fractions. [4]

2. Using the substitution $u = \sin^2 x$, evaluate the integral

$$\int_0^{\frac{\pi}{2}} \frac{\sin 2x}{\sqrt{4 - \sin^4 x}} dx.$$

Give your answer in the form $\frac{\pi}{k}$, where k is a positive integer. [5]

3. The function f is defined by

$$\begin{aligned} f(x) &= e^{2x} && \text{for } x < 0, \\ f(x) &= (x + 1)^2 && \text{for } x \geq 0. \end{aligned}$$

Determine whether or not

- (a) f is continuous when $x = 0$, [3]
 (b) the derivative f' is continuous when $x = 0$. [3]

4. The complex number z is given by $1 + i\sqrt{3}$.

- (a) Find the modulus and the argument of z . [2]
 (b) Find the three cube roots of z , giving your answers in the form $x + iy$ with x and y correct to three decimal places. [6]

5. Find the general solution to the equation

$$\sin \theta + \sin 5\theta = \cos 2\theta. \quad [8]$$

6. Using de Moivre's Theorem, show that for $\sin \theta \neq 0$,

$$\frac{\sin 6\theta}{\sin \theta} = a \cos^5 \theta + b \cos^3 \theta + c \cos \theta,$$

where a, b, c are constants whose values are to be determined.

Hence determine the limiting value of $\frac{\sin 6\theta}{\sin \theta}$ as θ tends to π . [8]

7. The ellipse E has equation

$$4x^2 + 9y^2 = 36.$$

- (a) Find

- (i) the eccentricity,
 (ii) the coordinates of the foci. [4]

- (b) (i) Show that the point $P(3\cos \theta, 2\sin \theta)$ lies on E for all values of θ .

- (ii) Show that the equation of the tangent to E at P is

$$3y \sin \theta + 2x \cos \theta = 6.$$

- (iii) This tangent meets the x -axis at R and the y -axis at S . The midpoint of RS is denoted by M . Determine the equation of the locus of M as θ varies. [11]

8. The function f is defined by

$$f(x) = \frac{(x+4)(x-2)}{(x-4)}.$$

- (a) Write down the coordinates of the points of intersection of the graph of f and the coordinate axes. [1]

- (b) Determine the equation of

- (i) the vertical asymptote on the graph of f ,
 (ii) the asymptote that is not parallel to a coordinate axis. [4]

- (c) Find the coordinates of the stationary points on the graph of f . [4]

- (d) Sketch the graph of f and its asymptotes. [3]

- (e) The set $S = [-7, 3]$. Determine

- (i) $f(S)$,
 (ii) $f^{-1}(S)$. [6]

END OF PAPER