

# OCR

Oxford Cambridge and RSA

## Friday 12 June 2015 – Morning

### A2 GCE MATHEMATICS

4723/01 Core Mathematics 3

#### QUESTION PAPER

Candidates answer on the Printed Answer Book.

**OCR supplied materials:**

- Printed Answer Book 4723/01
- List of Formulae (MF1)

**Other materials required:**

- Scientific or graphical calculator

**Duration:** 1 hour 30 minutes



#### INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

#### INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

#### INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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1 Find the equation of the tangent to the curve  $y = \frac{5x+4}{3x-8}$  at the point  $(2, -7)$ . [5]

2 It is given that  $\theta$  is the acute angle such that  $\cot \theta = 4$ . Without using a calculator, find the exact value of

(i)  $\tan(\theta + 45^\circ)$ , [3]

(ii)  $\operatorname{cosec} \theta$ . [2]

3 The volume,  $V$  cubic metres, of water in a reservoir is given by

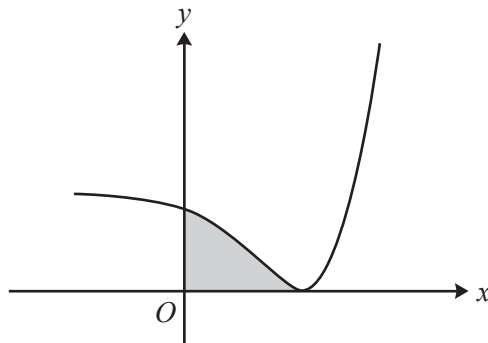
$$V = 3(2 + \sqrt{h})^6 - 192,$$

where  $h$  metres is the depth of the water. Water is flowing into the reservoir at a constant rate of 150 cubic metres per hour. Find the rate at which the depth of water is increasing at the instant when the depth is 1.4 metres. [5]

4 It is given that  $|x + 3a| = 5a$ , where  $a$  is a positive constant. Find, in terms of  $a$ , the possible values of

$$|x + 7a| - |x - 7a|. \quad [6]$$

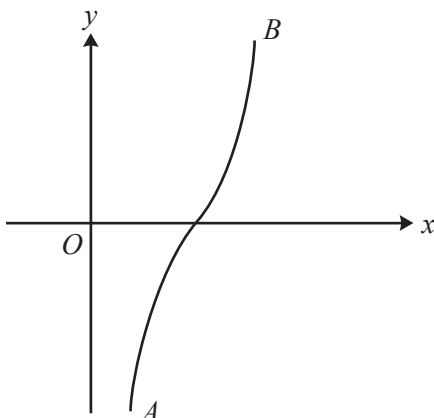
5



The diagram shows the curve  $y = e^{3x} - 6e^{2x} + 32$ .

(i) Find the exact  $x$ -coordinate of the minimum point and verify that the  $y$ -coordinate of the minimum point is 0. [4]

(ii) Find the exact area of the region (shaded in the diagram) enclosed by the curve and the axes. [4]



The diagram shows the curve  $y = 8 \sin^{-1}\left(x - \frac{3}{2}\right)$ . The end-points  $A$  and  $B$  of the curve have coordinates  $(a, -4\pi)$  and  $(b, 4\pi)$  respectively.

(i) State the values of  $a$  and  $b$ . [2]

(ii) It is required to find the root of the equation  $8 \sin^{-1}\left(x - \frac{3}{2}\right) = x$ .

(a) Show by calculation that the root lies between 1.7 and 1.8. [3]

(b) In order to find the root, the iterative formula

$$x_{n+1} = p + \sin(qx_n),$$

with a suitable starting value, is to be used. Determine the values of the constants  $p$  and  $q$  and hence find the root correct to 4 significant figures. Show the result of each step of the iteration process. [5]

7 (i) Find the exact value of  $\int_1^9 (7x+1)^{\frac{1}{3}} dx$ . [4]

(ii) Use Simpson's rule with two strips to show that an approximate value of  $\int_1^9 (7x+1)^{\frac{1}{3}} dx$  can be expressed in the form  $m + n \sqrt[3]{36}$ , where the values of the constants  $m$  and  $n$  are to be stated. [3]

(iii) Use the results from parts (i) and (ii) to find an approximate value of  $\sqrt[3]{36}$ , giving your answer in the form  $\frac{p}{q}$  where  $p$  and  $q$  are integers. [2]

**Question 8 begins on page 4.**

8 The functions  $f$  and  $g$  are defined as follows:

$$f(x) = 2 + \ln(x+3) \text{ for } x \geq 0,$$

$$g(x) = ax^2 \text{ for all real values of } x, \text{ where } a \text{ is a positive constant.}$$

(i) Given that  $gf(e^4 - 3) = 9$ , find the value of  $a$ . [3]

(ii) Find an expression for  $f^{-1}(x)$  and state the domain of  $f^{-1}$ . [3]

(iii) Given that  $ff(e^N - 3) = \ln(53e^2)$ , find the value of  $N$ . [5]

9 It is given that  $f(\theta) = \sin(\theta + 30^\circ) + \cos(\theta + 60^\circ)$ .

(i) Show that  $f(\theta) = \cos \theta$ . Hence show that

$$f(4\theta) + 4f(2\theta) \equiv 8 \cos^4 \theta - 3. \quad [6]$$

(ii) Hence

(a) determine the greatest and least values of  $\frac{1}{f(4\theta) + 4f(2\theta) + 7}$  as  $\theta$  varies, [3]

(b) solve the equation

$$\sin(12\alpha + 30^\circ) + \cos(12\alpha + 60^\circ) + 4 \sin(6\alpha + 30^\circ) + 4 \cos(6\alpha + 60^\circ) = 1$$

$$\text{for } 0^\circ < \alpha < 60^\circ. \quad [4]$$

**END OF QUESTION PAPER**

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