

Thursday 14 May 2015 – Morning

AS GCE MATHEMATICS

4725/01 Further Pure Mathematics 1

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:

- Printed Answer Book 4725/01
- List of Formulae (MF1)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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- 1 The complex number $x + iy$ is denoted by z . Express $3zz^* - |z|^2$ in terms of x and y . [3]
- 2 Find $\sum_{r=1}^n (3r^2 - 5)$, expressing your answer in a fully factorised form. [4]
- 3 The matrix \mathbf{A} is given by $\mathbf{A} = \begin{pmatrix} 2 & a \\ 0 & 1 \end{pmatrix}$, where a is a constant.
- (i) Find \mathbf{A}^{-1} . [2]
- The matrix \mathbf{B} is given by $\mathbf{B} = \begin{pmatrix} 2 & a \\ 4 & 1 \end{pmatrix}$.
- (ii) Given that $\mathbf{PA} = \mathbf{B}$, find the matrix \mathbf{P} . [3]
- 4 Prove by induction that, for $n \geq 1$, $\sum_{r=1}^n r(3r+1) = n(n+1)^2$. [5]
- 5 The loci C_1 and C_2 are given by $|z+2| = 2$ and $\arg(z+2) = \frac{5}{6}\pi$ respectively.
- (i) Sketch, on a single Argand diagram, the loci C_1 and C_2 . [4]
- (ii) Find the complex number represented by the intersection of C_1 and C_2 . [2]
- (iii) Indicate, by shading, the region of the Argand diagram for which
- $$|z+2| \leq 2 \text{ and } \frac{5}{6}\pi \leq \arg(z+2) \leq \pi. \quad [2]$$
- 6 The matrix \mathbf{M} is given by $\mathbf{M} = \begin{pmatrix} 0 & 2 \\ -1 & 0 \end{pmatrix}$.
- (i) The diagram in the Printed Answer Book shows the unit square $OABC$. The image of the unit square under the transformation represented by \mathbf{M} is $OA'B'C'$. Draw and label $OA'B'C'$, indicating clearly the coordinates of A' , B' and C' . [3]
- (ii) The transformation represented by \mathbf{M} is equivalent to a transformation P followed by a transformation Q . Give geometrical descriptions of a possible pair of transformations P and Q and state the matrices that represent them. [4]
- 7 (i) Use an algebraic method to find the square roots of the complex number $5 + 12i$. You must show sufficient working to justify your answers. [5]
- (ii) Hence solve the quadratic equation $x^2 - 4x - 1 - 12i = 0$. [5]
- 8 (i) Show that $\frac{3}{r-1} - \frac{2}{r} - \frac{1}{r+1} \equiv \frac{4r+2}{r(r^2-1)}$. [2]
- (ii) Hence find an expression, in terms of n , for $\sum_{r=2}^n \frac{4r+2}{r(r^2-1)}$. [6]
- (iii) Hence find the value of $\sum_{r=4}^{\infty} \frac{4r+2}{r(r^2-1)}$. [2]

9 The matrix \mathbf{D} is given by $\mathbf{D} = \begin{pmatrix} 1 & 3 & 4 \\ 2 & a & 3 \\ 0 & 1 & a \end{pmatrix}$.

(i) Find the values of a for which \mathbf{D} is singular.

[6]

(ii) Three simultaneous equations are shown below.

$$\begin{aligned} x + 3y + 4z &= 3 \\ 2x + ay + 3z &= 2 \\ y + az &= 0 \end{aligned}$$

For each of the following values of a , determine whether or not there is a unique solution. If a unique solution does not exist, determine whether the equations are consistent or inconsistent.

(a) $a = 3$

(b) $a = 1$

[4]

10 The cubic equation $x^3 + 4x + 3 = 0$ has roots α , β and γ .

(i) Use the substitution $x = \sqrt{u}$ to obtain a cubic equation in u .

[3]

(ii) Find the value of $\alpha^4 + \beta^4 + \gamma^4 + \alpha\beta\gamma$.

[7]

END OF QUESTION PAPER

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