

GCE

Mathematics

Unit **4730**: Mechanics 3

Advanced GCE

Mark Scheme for June 2015

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Answer		Marks	Guidance		
1	(i)	impulse momentum diagram	M1 A1	right-angled triangle with angle α and sides labelled 3, v and $I/0.2$ or 0.6, $0.2v$ and I	correct orientation, α and one side labelled correctly, right angle implied first two marks may be implied by correct working
		$\tan \alpha = I/(0.2 \times 3)$ $I = 0.25$ shown OR $0.2 \times 3 = 0.2v \cos \alpha$ and $I = 0.2v \sin \alpha$ $\frac{I}{0.2 \times 3} = \tan \alpha$ $I = 0.25$	M1 A1 [4]	AG	
	(ii)	$\cos \alpha = 3/v$ (speed) = 3.25 m s^{-1}	M1 A1 [2]	or using Pythagoras, with 3 and 1.25 oe	
2	m	Moments about B for BC $75L \cos \beta = 50 \times 2L \sin \beta$ $\tan \beta = 3/4$	M1 A1 A1 [3]	2 terms involving $\sin \beta$ and $\cos \beta$, 75 and 50	allow sin/cos error/ sign error allow missing L
	(ii)	moments about A for both rods $WL \cos \alpha + 75(2L \cos \alpha + L \cos \beta) =$ $50(2L \sin \alpha + 2L \sin \beta)$ correct values for $\sin/\cos \alpha/\beta$ attempt to solve ($W=$) 90 (N) OR ‘ X ’ = 50 N to right on AB oe ‘ Y ’ = 75 N down on AB oe Moments about A for AB $WL \cos \alpha + 75 \times 2L \cos \alpha = 50 \times 2L \sin \alpha$ ($W=$) 90 (N)	M1* A1 B1 *M1 A1 [5] B1 B1 M1 A1 A1	all (5) terms present; each term involves $\sin/\cos \alpha/\beta$. Dim correct: no extra terms dep M1A1 dep B1 also sc B1 for magnitudes if directions wrong/missing involves W , 75, 50, $\sin \alpha$ and $\cos \alpha$. dimensionally correct; no extra terms with substitution for α	allow sin/cos, $L/2L$, sign errors L may be cancelled all 4 seen all values substituted 50 & 75 may be seen on diagram in (i) L may be cancelled

		Answer	Marks	Guidance	
3	(i)	use of $T = \frac{\lambda x}{l}$ $T = \frac{10 \times 0.2}{0.4} + \frac{12 \times 0.1}{0.5}$ $W = 7.4 \text{ N}$ use of $E = \frac{\lambda x^2}{2l}$ $E = \frac{10(0.2)^2}{2 \times 0.4} + \frac{12(0.1)^2}{2 \times 0.5}$ $E = 0.62 \text{ (J)}$	M1 A1 A1 M1 A1 A1 [6]	used at least once CAO AG used at least once may see 0.5 + 0.12	
	(ii)	use of $F = ma$ when further extension is x $7.4 - \frac{10 \times (x + 0.2)}{0.4} - \frac{12 \times (x + 0.1)}{0.5} = \frac{7.4}{g} a$ $a = -\frac{49g}{7.4} x$ SHM: $\omega^2 = \frac{49g}{7.4}$ (or $\frac{2401}{37}$ or 64.89189) Use of $T = \frac{2\pi}{\omega}$ period is 0.780 (secs) $\frac{2\pi\sqrt{37}}{49}$ all subsequent motion is SHM because string does not become slack	M1* A1 A1 A1 *M1 A1 B1 [7]	allow sign errors, 'm' wrong 'F' correct accept $a = -64.89\dots x$, $a = -\frac{2401}{37} x$ oe dep on all first 3 marks must subst for their ω allow if ω correct justified at some point	OR, when total length of string is x $7.4 - \frac{10 \times (x - 0.4)}{0.4} - \frac{12 \times (x - 0.5)}{0.5} = \frac{7.4}{g} a$ $a = -\frac{49g}{7.4} (x - 0.6)$ SHM about $x = 0.6$, and ω^2 given 0.77998
4	(i)	$-\frac{v}{8} = 0.4 \frac{dv}{dt}$ $t = -3.2 \int \frac{1}{v} dv$ $t = -3.2 \ln v + 3.2 \ln 10$ time taken = 3.2ln2 or 2.22 (s)	M1* A1 *M1 A1 A1 [5]	allow sign error, allow 0.4a attempt to separate variables and integrate or $t = -3.2 \int_{10}^5 \frac{1}{v} dv$ $t = -3.2 \ln v$; limits used correctly 2.21807...	

	Answer	Marks	Guidance	
	<p>(ii)</p> $-\frac{v}{8} = 0.4v \frac{dv}{dx}$ $x = -3.2 \int dv$ $x = -3.2v + 32$ <p>ave speed = $x/(i)$ ave speed = 7.21</p> <p>OR</p> $\frac{dx}{dt} = 10e^{-\frac{t}{3.2}}$ $x = 10 \int e^{-\frac{t}{3.2}} dt$ $x = 32 \left(1 - e^{-\frac{t}{3.2}}\right)$ <p>ave speed = $x/(i)$ ave speed = 7.21</p>	<p>M1*</p> <p>A1</p> <p>*M1</p> <p>A1</p> <p>*M1</p> <p>A1 [6]</p> <p>M1*</p> <p>A1</p> <p>*M1</p> <p>A1</p> <p>*M1</p> <p>A1</p>	<p>allow sign error</p> <p>attempt to separate variables and integrate</p> <p>$x = 16$ when $v = 5$.</p> <p>for M1, ft from (i), must contain ln term</p> <p>attempt to separate variables and integrate</p> <p>must show constant or use limits correctly</p> <p>dep all 5 previous marks</p>	<p>their x evaluated accept 5/ln2</p> <p>$x = 16$ when $t = 3.2 \ln(2)$</p> <p>accept 5/ln2</p>
5	<p>(i)</p> <p>use of conservation of momentum $2m\cos\alpha - mbc\cos\beta = mx_2x\cos 45^\circ$ use of NEL</p> <p>$2\cos 45^\circ - 0 = -2/3 (-bc\cos\beta - a\cos\alpha)$ attempt to eliminate $a\cos\alpha$ or $bc\cos\beta$ $a\cos\alpha = 5\sqrt{2}/6$ $bc\cos\beta = 2\sqrt{2}/3$ oe</p>	<p>M1*</p> <p>A1</p> <p>M1*</p> <p>A1</p> <p>*M1</p> <p>A1</p> <p>A1 [7]</p>	<p>must be 3 non-zero terms</p> <p>must be 3 non-zero terms, and 'e' in correct position</p> <p>dep both previous M1 marks</p> <p>AG dep final M1 and www</p>	<p>allow sign errors, $m/2m$ errors, sin/cos</p> <p>allow sign errors, sin/cos,</p>
	<p>(ii)</p> <p>$a\sin\alpha = 2$ attempt to solve $a\sin\alpha = 2$ and $a\cos\alpha = 5\sqrt{2}/6$ $a = 2.32$ $\alpha = 59.5^\circ$</p>	<p>B1</p> <p>M1</p> <p>A1</p> <p>A1 [4]</p>	<p>need to eliminate a or α</p> <p>accept 1.03 radians</p>	<p>2.321398..., 59.49104...°, 1.0383...rad</p>

Answer		Marks	Guidance
6	(i)	$a = 0.6 \text{ (m)}$ $\omega = 4$ $\text{max vel} = a\omega = 2.4 \text{ (m s}^{-1}\text{)}$	B1 B1 B1ft[3] accept sight of $\frac{\pi}{0.25\pi}$ or $\frac{2\pi}{0.5\pi}$ ft from wrong a and/or ω
	(ii)	<i>must use their a and ω from (i) unless defined differently in (ii)</i> $x = 0.6\cos 4 \times 0.7$ $x = -0.565$ $v = -0.6 \times 4 \times \sin 4 \times 0.7$ $v = -0.804$	M1 A1 M1 A1 [4] use of $a\cos\omega t$; complete method use of $(-)\omega\sin\omega t$ or $v^2 = \omega^2(a^2 - x^2)$ if v^2 formula used, direction of v needs to be made clear.
	(iii)	<i>do not accept answers from wrong working</i> t and x for one point t and x for second point correctly giving precisely 2 other occasions, with x and t values matching sc, if < 3 scored, both t values B2 or one t value B1 or $x = 0.565$ B1 of B0 scored allow B1 for number of other occasions shown to be 2	B2 B1 B1 values of t are = 0.0854, 0.871 values of x are 0.565, -0.565 dep first 3 marks ignore wrong values “ “ [4]

$\pi/4 - 0.7, \pi/2 - 0.7$
 ignore ref to point when $t = 0.7$
 can show on diagram

 P has this speed 4 times in 1 period
 (1.570 s) so 2 other times in $0 < t < 1$

		Answer	Marks	Guidance
7	(i)	using $F = ma$ $T - 0.2g\cos\theta = 0.2v^2/0.5$ by energy $\frac{1}{2} \times 0.2u^2 = \frac{1}{2} \times 0.2v^2 + 0.2g \times 0.5(1 - \cos\theta)$ $T = 5.88\cos\theta + 0.4u^2 - 3.92$	M1 A1 M1 A1 A1 [5]	must have the right 3 terms; allow sign error / sin for cos for M1 $v^2 = u^2 - 9.8(1 - \cos\theta)$ AG with no errors and no gaps in argument
	(ii)	when $\theta = 180^\circ$, $5.88\cos\theta + 0.4u^2 - 3.92 = 0$ $-5.88 + 0.4u^2 - 3.92 = 0$ min u is 4.95 (m s ⁻¹) OR, at top, $mg = \frac{mv^2}{r}$, so $v^2 = 0.5g$ by energy $\frac{1}{2} \times 0.2u^2 = \frac{1}{2} \times 0.2 \times 0.5g + 0.2g$ min u is 4.95 (m s ⁻¹)	M1 A1 A1 [3] B1 M1 A1	allow inequalities for M1A1 $\frac{7}{2}\sqrt{2}$ allow inequalities for B1M1 4.9497... Not > 4.95
	(iii)	$5.88\cos\theta + 0.4 \times 12.25 - 3.92 = 0$ $\cos\theta = (3.92 - 4.9)/5.88$ (= -1/6) use energy eq ⁿ from (i) $\frac{1}{2} \times 0.2 \times 3.5^2 = \frac{1}{2} \times 0.2v^2 + 0.2g \times 0.5(1 - \cos\theta)$ $v = 0.904 \text{ m s}^{-1}$ OR use T equation from (i) $0 - 0.2g(-1/6) = 0.2v^2/0.5$ $v = 0.904 \text{ m s}^{-1}$	M1 A1 M1 A1 [4] M1 A1	might see $\theta = 99.6^\circ$ or 1.74 radians accept use of their θ $\frac{7}{30}\sqrt{15}$ 99.49406...°, 1.73824...rads 0.903696...

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