OXFORD CAMBRIDGE AND RSA EXAMINATIONS
A2 GCE

4724/01

MATHEMATICS

Core Mathematics 4

QUESTION PAPER

WEDNESDAY 18 JUNE 2014: Afternoon
DURATION: 1 hour 30 minutes
plus your additional time allowance

MODIFIED ENLARGED

Candidates answer on the Printed Answer Book or any suitable paper provided by the centre.
The Printed Answer Book may be enlarged by the centre.

OCR SUPPLIED MATERIALS:

Printed Answer Book 4724/01
List of Formulae (MF1)

OTHER MATERIALS REQUIRED:

Scientific or graphical calculator

READ INSTRUCTIONS OVERLEAF

DC (CW/CGW) 4060449433 100069/1

INSTRUCTIONS TO CANDIDATES

Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book or on the paper provided by the centre. Please write clearly and in capital letters.

IF YOU USE THE PRINTED ANSWER BOOK, WRITE YOUR ANSWER TO EACH QUESTION IN THE SPACE PROVIDED. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).

Use black ink. HB pencil may be used for graphs and diagrams only.

Answer ALL the questions.

Read each question carefully. Make sure you know what you have to do before starting your answer.

You are permitted to use a scientific or graphical calculator in this paper.

Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.

You are reminded of the need for clear presentation in your answers.

The total number of marks for this paper is 72.

Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

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1 Express $x + \frac{1}{1-x} + \frac{2}{1+x}$ as a single fraction, simplifying your answer. [3]

2 The points $O(0, 0, 0)$, $A(2, 8, 2)$, $B(5, 5, 8)$ and $C(3, -3, 6)$ form a parallelogram $OABC$. Use a scalar product to find the acute angle between the diagonals of this parallelogram. [5]

3 (i) Find the first three terms in the expansion of $(1 - 2x)^{-\frac{1}{2}}$ in ascending powers of x, where $|x| < \frac{1}{2}$. [3]

(ii) Hence find the coefficient of x^2 in the expansion of $\frac{x+3}{\sqrt{1-2x}}$. [2]

4 Show that $\int_0^{\frac{1}{4}\pi} \frac{1 - 2\sin^2 x}{1 + 2\sin x \cos x} \, dx = \frac{1}{2}\ln 2$. [5]

5 The equations of three lines are as follows.

Line A: $\mathbf{r} = \mathbf{i} + 4\mathbf{j} + \mathbf{k} + s(-\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$

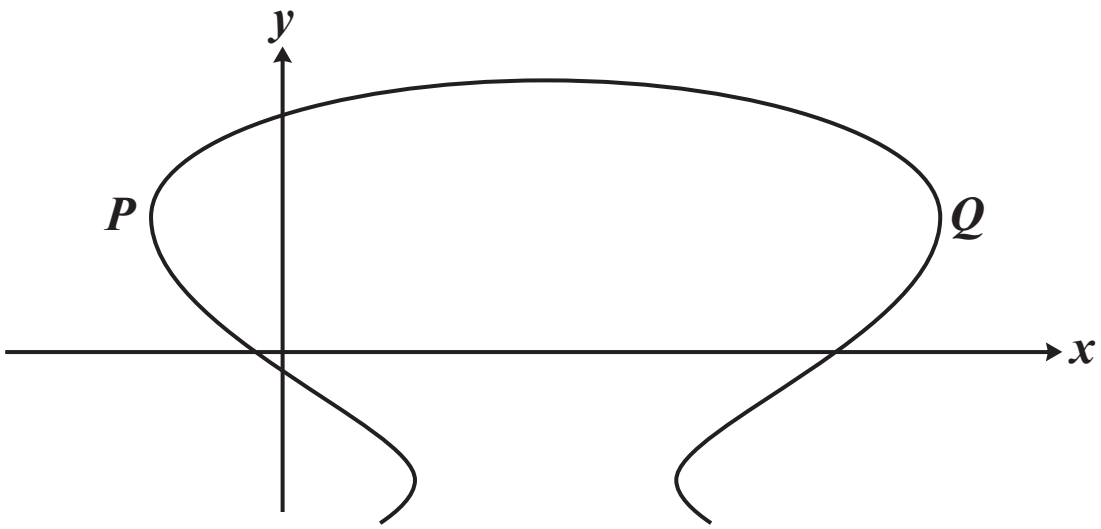
Line B: $\mathbf{r} = 2\mathbf{i} + 8\mathbf{j} + 2\mathbf{k} + t(\mathbf{i} + 3\mathbf{j} + 5\mathbf{k})$

Line C: $\mathbf{r} = -\mathbf{i} + 19\mathbf{j} + 15\mathbf{k} + u(2\mathbf{i} - 4\mathbf{j} - 4\mathbf{k})$

(i) Show that lines A and B are skew. [4]

(ii) Determine, giving reasons, the geometrical relationship between lines A and C. [2]

6 The diagram below shows the curve with equation $x^2 + y^3 - 8x - 12y = 4$.



At each of the points P and Q the tangent to the curve is parallel to the y-axis. Find the coordinates of P and Q. [8]

7 A curve has parametric equations

$$x = 2\sin t, \quad y = \cos 2t + 2\sin t$$

for $-\frac{1}{2}\pi \leqslant t \leqslant \frac{1}{2}\pi$.

(i) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = 1 - 2\sin t$ and hence find the coordinates of the stationary point. [5]

(ii) Find the cartesian equation of the curve. [3]

(iii) State the set of values that x can take and hence sketch the curve. [3]

8 (i) Use division to show that

$$\frac{t^3}{t+2} \equiv t^2 - 2t + 4 - \frac{8}{t+2}.$$ [3]

(ii) Find $\int_1^2 6t^2 \ln(t+2)\,\mathrm{d}t$. Give your answer in the form $A + B\ln 3 + C\ln 4$. [6]

9 Express $\frac{2+x^2}{(1+2x)(1-x)^2}$ in partial fractions and hence show that $\int_0^{\frac{1}{4}} \frac{2+x^2}{(1+2x)(1-x)^2}\mathrm{d}x = \frac{1}{2}\ln\frac{3}{2} + \frac{1}{3}$. [9]

10 A container in the shape of an inverted cone of radius 3 metres and vertical height 4.5 metres is initially filled with liquid fertiliser. This fertiliser is released through a hole in the bottom of the container at a rate of 0.01 m^3 per second. At time t seconds the fertiliser remaining in the container forms an inverted cone of height h metres.

[The volume of a cone is $V = \frac{1}{3}\pi r^2 h$.]

(i) Show that $h^2\frac{\mathrm{d}h}{\mathrm{d}t} = -\frac{9}{400\pi}$. [5]

(ii) Express h in terms of t. [4]

(iii) Find the time it takes to empty the container, giving your answer to the nearest minute. [2]

END OF QUESTION PAPER

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