Rewarding Learning

ADVANCED
General Certificate of Education
2019

Mathematics

Assessment Unit A2 2

assessing

Applied Mathematics

[AMT21]

WEDNESDAY 5 JUNE, MORNING

MARK SCHEME

General Marking Instructions

GCE Advanced/Advanced Subsidiary (AS) Mathematics

Introduction

The mark scheme normally provides the most popular solution to each question. Other solutions given by candidates are evaluated and credit given as appropriate; these alternative methods are not usually illustrated in the published mark scheme.

The marks awarded for each question are shown in the right-hand column and they are prefixed by the letters **M**, **W** and **MW** as appropriate. The key to the mark scheme is given below:

M indicates marks for correct method.

W indicates marks for working.

MW indicates marks for combined method and working.

The solution to a question gains marks for correct method and marks for accurate working based on this method. Where the method is not correct no marks can be given.

A later part of a question may require a candidate to use an answer obtained from an earlier part of the same question. A candidate who gets the wrong answer to the earlier part and goes on to the later part is naturally unaware that the wrong data is being used and is actually undertaking the solution of a parallel problem from the point at which the error occurred. If such a candidate continues to apply correct method, then the candidate's individual working must be followed through from the error. If no further errors are made, then the candidate is penalised only for the initial error. Solutions containing two or more working or transcription errors are treated in the same way. This process is usually referred to as "follow-through marking" and allows a candidate to gain credit for that part of a solution which follows a working or transcription error.

Positive marking

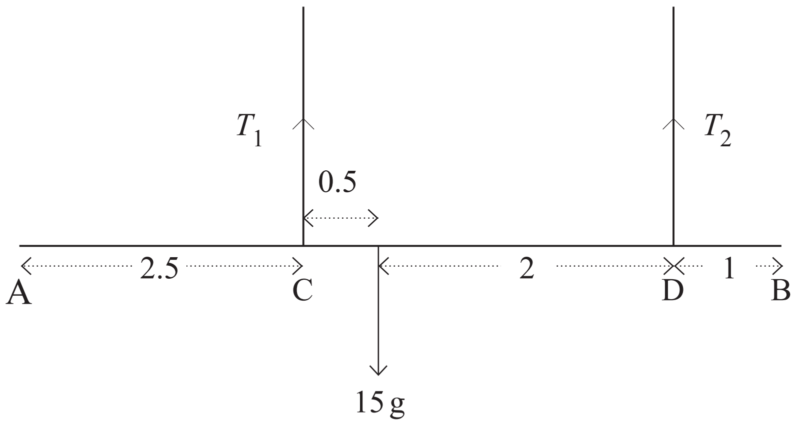
It is our intention to reward candidates for any demonstration of relevant knowledge, skills or understanding. For this reason we adopt a policy of following through their answers, that is, having penalised a candidate for an error, we mark the succeeding parts of the question using the candidate's value or answers and award marks accordingly.

Some common examples of this occur in the following cases:

(a) a numerical error in one entry in a table of values might lead to several answers being incorrect, but these might not be essentially separate errors;

(b) readings taken from a candidate's inaccurate graphs may not agree with the answers expected but might be consistent with the graphs drawn.

When the candidate misreads a question in such a way as to make the question easier only a proportion of the marks will be available (based on the professional judgement of the examining team).

			AVAILABLE MARKS
1	**(i)** 	MW1	
	(ii) Take moments about C		
	Clockwise moment about C = Anti-clockwise moment about C		
	$15\,g \times 0.5 = T_2 \times 2.5$	M1 M1	
	$T_2 = 29.4\,N$	W1	
	R(↑): Equilibrium		
	$T_1 + T_2 = 15\,g$	MW1	
	$T_1 = 15\,g - 29.4$		
	$T_1 = 117.6\,N$	W1	
	$T_1 = 118\,N$ (3 sf)		6
2	**(i)** Momentum before = Momentum after		
	$2u(6m) + 3m(-6u) = (-2u)(6m) + (ku)(3m)$	M1 M1 W1	
	$12mu - 18mu = -12mu + 3kmu$		
	$6mu = 3kmu$		
	$k = 2$	MW1	
	(ii) Impulse = Change in Momentum		
	$I = mv - mu$	M1	
	$I = 3m(2u - (-6u))$		
	$I = 24mu$ Ns in opposite direction to the initial direction of T	MW2	7

			AVAILABLE MARKS
3	**(i)** When $v = 0$, $0 = 2t^2 - 9t + 4$ $0 = (2t - 1)(t - 4)$	M1	
	$t = \frac{1}{2}$ or $t = 4$	MW1 MW1	
	(ii) $a = \frac{\mathrm{d}v}{\mathrm{d}t}$	M1	
	$a = 4t - 9$	MW2	
	When $t = 3$ $a = 4(3) - 9 = 3\ \mathrm{m\,s^{-2}}$	MW1	
	(iii) STOPS TWICE		
	$s = \int v \ \mathrm{d}t$	M1	
	$s = \frac{2}{3}t^3 - \frac{9}{2}t^2 + 4t + c$	W1	
	$t = 0$, $s = 0$ therefore $c = 0$	MW1	
	$t = \frac{1}{2}, \quad s = \frac{2}{3}\left(\frac{1}{2}\right)^3 - \frac{9}{2}\left(\frac{1}{2}\right)^2 + 4\left(\frac{1}{2}\right) = \frac{23}{24}$	M1	
	$t = 4, \quad s = \frac{2}{3}(4)^3 - \frac{9}{2}(4)^2 + 4(4) = -13\frac{1}{3}$	MW1	
	$t = 5, \quad s = \frac{2}{3}(5)^3 - \frac{9}{2}(5)^2 + 4(5) = -9\frac{1}{6}$	MW1	
	Therefore, distance:		
	$d = \frac{23}{24} + \left(\frac{23}{24} + 13\frac{1}{3}\right) + \left(13\frac{1}{3} - 9\frac{1}{6}\right) = 19\frac{5}{12}$ or 19.4 m	M1 W1	15

AVAILABLE MARKS

4 **(i)** **Vert.**

$u = u \sin \alpha$ $\quad\quad s = ut + \frac{1}{2}at^2$

$a = -g$ $\quad\quad y = uT \sin \alpha - \frac{g}{2}T^2$ M1 W1

$t = T$

$s = y$

Horiz.

$s = x$ $\quad\quad s = ut + \frac{1}{2}at^2$ M1

$u = u \cos \alpha$ $\quad\quad x = uT \cos \alpha$

$t = T$ $\quad\quad T = \frac{x}{u \cos \alpha}$ MW1

$a = 0$

Sub T into y

$y = u\left(\frac{x}{u \cos \alpha}\right) \sin \alpha - \frac{g}{2}\left(\frac{x}{u \cos \alpha}\right)^2$ M1 W1

$y = x \tan \alpha - \frac{gx^2}{2u^2}\left(\frac{1}{\cos^2 \alpha}\right)$

$y = x \tan \alpha - \frac{gx^2}{2u^2}\ (1 + \tan^2 \alpha)$ MW1

(ii) $u = 14, y = 0, x = 15, \alpha = ?$

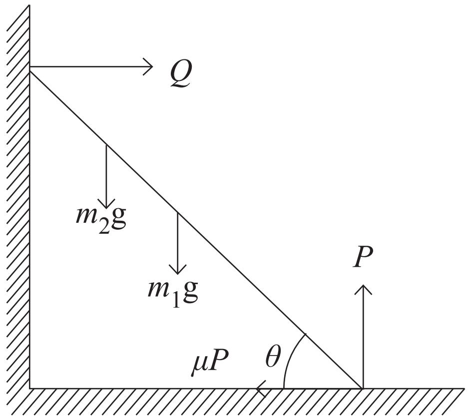
$0 = 15 \tan \alpha - \frac{g(15)^2}{2(14)^2}(1 + \tan^2 \alpha)$ M1 W1

$0 = 3 \tan^2 \alpha - 8 \tan \alpha + 3$ MW1

$\tan \alpha = 2.215\ldots$ *OR* $\tan \alpha = 0.4514\ldots$

$\alpha = 65.7°$ *OR* $\alpha = 24.3°$ MW1 MW1 12

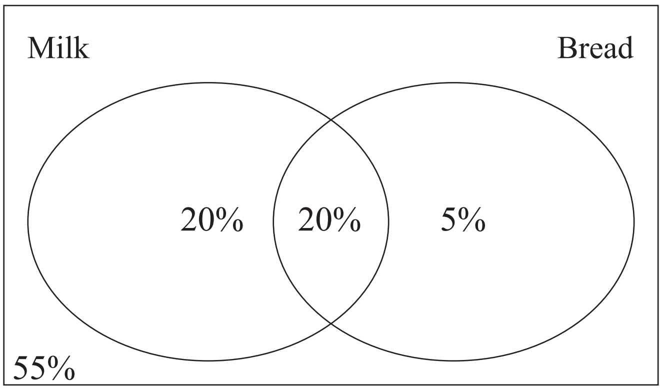
5



$\sin\theta = \dfrac{12}{13}$ $\cos\theta = \dfrac{5}{13}$

		AVAILABLE MARKS
R(↑) $P = (m_1 + m_2)$g	MW1	
R(→) $Q = \mu P$ $Q = \mu$g$(m_1 + m_2)$	M1 W1 MW1	
Take moments about foot of the ladder. Clockwise moment = Anti-clockwise moment		
$6Q \sin\theta = 3m_1\text{g}\cos\theta + xm_2\text{g}\cos\theta$	M1 MW2	
$6\mu \sin\theta\,(m_1 + m_2) - 3m_1 \cos\theta = xm_2 \cos\theta$	MW1	
$x = \dfrac{72\mu(m_1 + m_2) - 15m_1}{5m_2}$ as required.	MW2	10

6 (a) (i)



AVAILABLE MARKS

40% + 25% + 55% = 120% M1

120% − 100% = 20%

P(bread and no milk) = 0.05 MW1

Alternative solution

$P(M \cup B) = P(M) + P(B) - P(M \cap B)$

$0.45 = 0.4 + 0.25 - P(M \cap B)$ M1

$P(M \cap B) = 0.2$

$P(B) = P(M \cap B) + P(M' \cap B)$

$0.25 = 0.2 + P(M' \cap B)$

$P(M' \cap B) = 0.05$ MW1

(ii) $P(B|M) = \dfrac{P(B \cap M)}{P(M)} = \dfrac{0.20}{0.40} = \dfrac{1}{2}$ M1 W1

Alternative solution

from Venn Diagram $\dfrac{20}{40} = \dfrac{1}{2}$ M1 W1

(iii) If two events are independent then $P(M \cap B) = P(M) \times P(B)$

$P(M \cap B) = 0.20$

$P(M) \times P(B) = 0.4 \times 0.25 = 0.1$ MW1

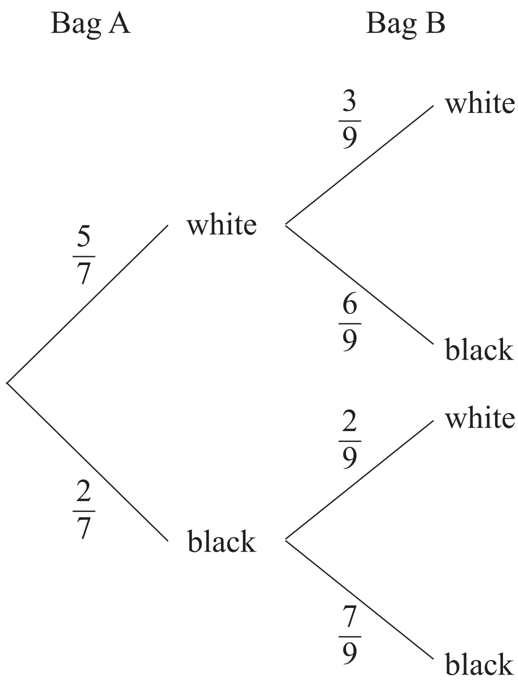
$0.20 \neq 0.1$ so the events are not independent MW1

Alternative solution

$P(\text{milk}|\text{bread}) = \dfrac{20}{25} = \dfrac{4}{5}$ MW1

$\dfrac{4}{5} \neq \dfrac{2}{5}$ therefore not independent MW1

(b) (i)



		AVAILABLE MARKS
$P(WW) = \frac{5}{7} \times \frac{3}{9} = \frac{5}{21}$	M1 W1	
(ii) $P(WW)$ or $P(BW) = \frac{5}{21} + \frac{2}{7} \times \frac{2}{9}$	MW1	
$= \frac{5}{21} + \frac{4}{63} = \frac{19}{63}$	W1	
(iii) $P(B_1 \mid W_2) = \frac{P(B_1 \cap W_2)}{P(W_2)} = \frac{\frac{4}{63}}{\frac{19}{63}} = \frac{4}{19}$	M1 W1 W1	13

7 (a) (i) $X \sim \mathrm{N}(\mu, \sigma^2)$

AVAILABLE MARKS

$1.175 = \dfrac{17.5 - \mu}{\sigma}$	M1 W1	
$1.175\sigma = 17.5 - \mu$	MW1	
$-0.674 = \dfrac{16.8 - \mu}{\sigma}$		
$-0.674\sigma = 16.8 - \mu$	MW1	
$1.175\,\sigma - 17.5 = -0.674\,\sigma - 16.8$	M1	
$\sigma = 0.37858$	W1	
$\mu = 17.055$ $= 17.1$ (3 sf)	W1	
Variance $= \sigma^2 = 0.143$ (3 sf)	MW1	
(ii) $\dfrac{17.2 - 17.055}{0.37858} = 0.383$	M1 W1	
$\mathrm{P}(X < 17.2) = 0.6491$	MW1	
$\mathrm{P}(X \geqslant 17.2) = 1 - 0.6491 = 0.3509$	M1	
$\Rightarrow 35.1\%$	W1	
(b) The normal distribution models many practical or natural distributions.	MW1	14

8	**(i)**	$X \sim B(10, 0.4)$	M1
		$1 - P(X = 0)$	M1
		$P(X = 0) = {}^{10}C_0(0.4)^0(0.6)^{10} = 0.0060466$	MW1
		$P(X \geqslant 1) = 0.99395$ $= 0.994$ (3 sf)	W1
	(ii)	H_0: p = 0.4	M1
		H_1: p > 0.4	M1
		$X \sim B(10, 0.4)$	
		one-tailed test, critical value 0.05	M1
		Reject H_0 if $P(X \geqslant 7)$ is less than 5%	
		$1 - P(X \leqslant 6)$	M1 W1
		$= 1 - 0.9452 = 0.0548$	W1
		$0.0548 > 5\%$	W1
		Alternative Solution	
		$P(X \geqslant 7) = 0.0548$ $P(X \geqslant 8) = 0.0123$	M1W1
		so critical region is 8, 9 and 10	W1
		Therefore 7 is not in critical region	W1
		There is insufficient evidence to reject the H_0 and accept H_1	MW1
		The teacher's belief that the new reading incentive improves the reading of the children is not supported by the evidence.	MW1
	(iii)	The test could be improved by the teacher taking a larger sample of children.	MW1

AVAILABLE MARKS: 14

				AVAILABLE MARKS
9	(i)	$X \sim N(28, 7.5^2)$		
		$H_0: \mu = 28$		
		$H_1: \mu \neq 28$	M1	
		two-tailed test (or diagram shown)	M1	
		z critical $= \pm 1.645$	MW1	
		Reject H_0 if $z < -1.645$, $z > 1.645$	MW1	
		$z = \dfrac{24.5-28}{\frac{7.5}{\sqrt{30}}}$	M1 W1	
		$= -2.556$	W1	
		Since $-2.556 < -1.645$ we reject H_0 and accept H_1		
		Conclude that there is sufficient evidence to show that the mean is different.	MW1	
	(ii)	Only the dedicated runners might take part in December because of the colder weather. So, possibly quicker times on average may occur.	MW1	9
			Total	**100**