Rewarding Learning

ADVANCED
General Certificate of Education
2017

Mathematics

Assessment Unit F2
assessing
Module FP2: Further Pure Mathematics 2

AMF21

[AMF21]

FRIDAY 16 JUNE, AFTERNOON

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all eight** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all eight questions.

Show clearly the development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 Find in terms of n

$$\sum_{r=1}^{n} (n-r)^2$$ [5]

2 **(i)** Show that

$$\tan x + \cot x \equiv 2 \operatorname{cosec} 2x$$ [3]

(ii) Hence or otherwise find, in radians, the general solution of the equation

$$\tan x + \cot x = 8 \cos 2x$$ [4]

3 **(i)** Express in partial fractions

$$f(x) = \frac{3x^2 + 1}{x(2x^2 + 1)}$$ [5]

(ii) Hence or otherwise find the exact value of

$$\int_1^2 \frac{3x^2+1}{x(2x^2+1)}\,dx$$

leaving your answer in the form $a \ln b$ [4]

4 Let

$$f(x) = e^{2x} \sin x$$

(i) Find $f'(x)$ [2]

(ii) Show that $f''(x) = 3e^{2x} \sin x + 4e^{2x} \cos x$ [1]

(iii) Find the Maclaurin expansion for $f(x) = e^{2x} \sin x$, up to and including the term in x^3 [5]

5 Using the principle of mathematical induction, prove that

$$\sum_{r=1}^{n} \frac{3r+2}{r(r+1)(r+2)} = \frac{n(2n+3)}{(n+1)(n+2)}$$ [7]

6 The distance x metres of a particle from the origin at time t seconds is given by the differential equation

$$2\frac{d^2x}{dt^2} + 3\omega\frac{dx}{dt} - 2\omega^2 x = \omega^2 e^{-\omega t}$$

where ω is a positive constant.

Given that $x = 1$ and $\frac{dx}{dt} = \omega$ when $t = 0$, find an expression for the distance x in terms of t. [12]

7 **(a)** A parabola may be defined as "the locus of a point which moves so that its distance from a fixed point (the focus) is equal to its perpendicular distance to a fixed line (the directrix)".

Given that the focus is $(a, 0)$ and the directrix has equation $x + a = 0$, deduce that the cartesian equation of this parabola is

$$y^2 = 4ax$$ [4]

(b) **(i)** Show that the equation

$$y^2 - 4y = 2x + 2$$

represents a parabola. [2]

(ii) Find the coordinates of the focus and the vertex and derive the equation of the directrix of this parabola. [3]

(iii) Sketch the parabola showing, with coordinates, the vertex and any points of intersection with the coordinate axes. [3]

8 Consider the complex number

$$\omega = \cos\frac{2\pi}{5} + \mathrm{i}\sin\frac{2\pi}{5}$$

(i) Find the value of ω^5 [2]

(ii) Prove that

$$1 + \omega + \omega^2 + \omega^3 + \omega^4 = 0$$ [3]

(iii) Write

$$(\omega + \omega^4)(\omega^2 + \omega^3)$$

in its simplest form. [3]

(iv) Derive a quadratic equation with integer coefficients which has roots $(\omega + \omega^4)$ and $(\omega^2 + \omega^3)$. [3]

(v) Hence show that

$$\cos\frac{2\pi}{5} = \frac{-1 + \sqrt{5}}{4}$$ [4]

THIS IS THE END OF THE QUESTION PAPER
