Rewarding Learning

ADVANCED
General Certificate of Education
2017

Mathematics

Assessment Unit M4
assessing
Module M4: Mechanics 4

[AMM41]

WEDNESDAY 28 JUNE, MORNING

MARK
SCHEME

Introduction

The mark scheme normally provides the most popular solution to each question. Other solutions given by candidates are evaluated and credit given as appropriate; these alternative methods are not usually illustrated in the published mark scheme.

The marks awarded for each question are shown in the right-hand column and they are prefixed by the letters **M**, **W** and **MW** as appropriate. The key to the mark scheme is given below:

M indicates marks for correct method.

W indicates marks for working.

MW indicates marks for combined method and working.

The solution to a question gains marks for correct method and marks for an accurate working based on this method. Where the method is not correct no marks can be given.

A later part of a question may require a candidate to use an answer obtained from an earlier part of the same question. A candidate who gets the wrong answer to the earlier part and goes on to the later part is naturally unaware that the wrong data is being used and is actually undertaking the solution of a parallel problem from the point at which the error occurred. If such a candidate continues to apply correct method, then the candidate's individual working must be followed through from the error. If no further errors are made, then the candidate is penalised only for the initial error. Solutions containing two or more working or transcription errors are treated in the same way. This process is usually referred to as "follow-through marking" and allows a candidate to gain credit for that part of a solution which follows a working or transcription error.

Positive marking:

It is our intention to reward candidates for any demonstration of relevant knowledge, skills or understanding. For this reason we adopt a policy of **following through** their answers, that is, having penalised a candidate for an error, we mark the succeeding parts of the question using the candidate's value or answers and award marks accordingly.

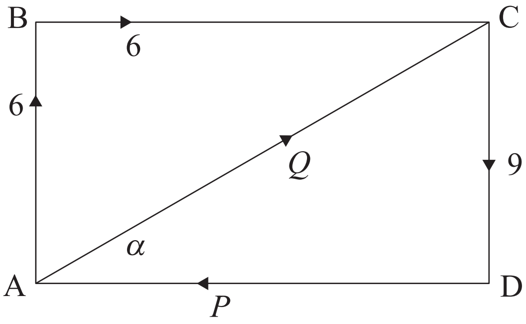
Some common examples of this occur in the following cases:

(a) a numerical error in one entry in a table of values might lead to several answers being incorrect, but these might not be essentially separate errors;

(b) readings taken from candidates' inaccurate graphs may not agree with the answers expected but might be consistent with the graphs drawn.

When the candidate misreads a question in such a way as to make the question easier only a proportion of the marks will be available (based on the professional judgement of the examining team).

1



AVAILABLE MARKS

(i) ↑ : $6 + Q\sin\alpha - 9 = 0$ M1 W1

$0.6Q = 3$

$Q = 5\text{N}$ W1

→ : $6 + Q\cos\alpha - P = 0$ M1 W1

$P = 6 + 0.8 \times 5 = 10\text{N}$ W1

(ii) Take moments about A: M1

$M = 6 \times 3 + 9 \times 4$

$M = 54\text{Nm}$ clockwise W2 9

2 **(i)** Centre of mass is $\frac{h}{4} = \frac{2l}{4} = \frac{l}{2}$ from centre of base. MW1

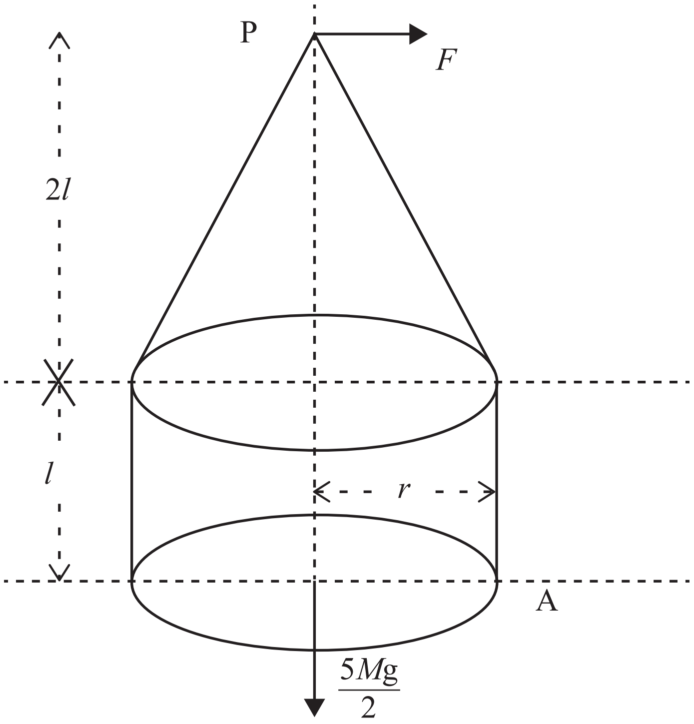
(ii) Volume of cone = $\frac{1}{3}\pi r^2 \times 2l = \frac{2}{3}\pi r^2 l$
Volume of cylinder = $\pi r^2 l$

$$\frac{\text{Mass of cylinder}}{\text{Mass of cone}} = \frac{\pi r^2 l}{\frac{2}{3}\pi r^2 l} = \frac{3}{2}$$ M1 W1

Mass of cylinder = $\frac{3M}{2}$

Mass of paperweight = $\frac{3M}{2} + M = \frac{5M}{2}$ MW1

(iii)



Take moments about A: $F \times 3l = \frac{5Mg}{2} \times r$ M1 W1

$$F = \frac{5r}{6l} \times Mg$$ W1 7

$$= \frac{5rMg}{6l}$$

3 **(i)** $mg = \frac{GMm}{R^2}$ M1

$G = \frac{gR^2}{M}$ W1

(ii) $[G] = [LT^{-2}][L]^2[M]^{-1}$ M1 W1

$= [L^3T^{-2}M^{-1}]$ W1

(iii) In orbit of radius *d* about the centre of the Earth, travelling at speed *v*:

$\frac{mv^2}{d} = \frac{GMm}{d^2}$ M2

$d = \frac{GM}{v^2}$ W1

From **(i)**, $GM = gR^2$

$d = \frac{gR^2}{v^2}$ MW1

$h = d - R = \frac{9.8 \times (6.44 \times 10^6)^2}{7500^2} - 6.44 \times 10^6$ M1

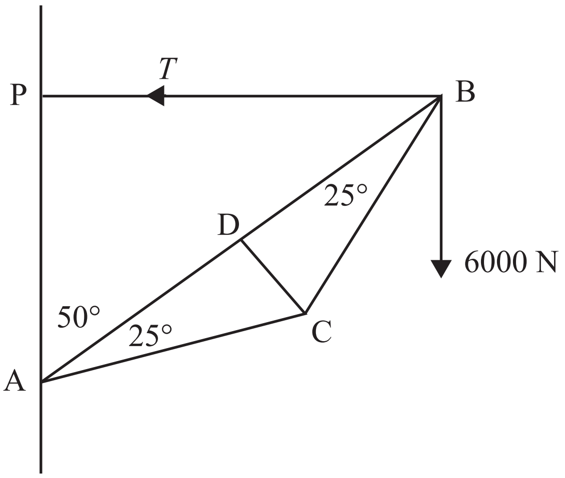
$= 7.86 \times 10^5$ m

$= 786$ km W1

11

4

AVAILABLE MARKS



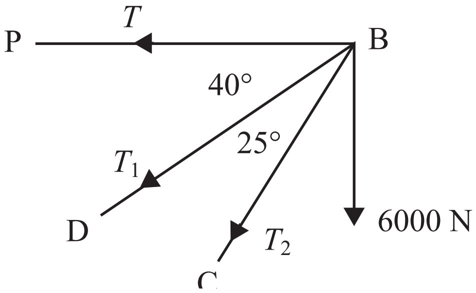
(i) Take moments about A: $6000 \times 12\sin 50° = T \times 12\cos 50°$ M1 W1

$T = 6000 \times \tan 50°$

$= 7150.521$ W1

$= 7150\text{N}$ (3 s.f.)

(ii) At B:



↖ : $T \times \sin 40° = T_2 \times \sin 25° + 6000 \times \sin 50°$ M1 W1

$T_2 \sin 25° = T\sin 40° - 6000\sin 50°$

$T_2 = 0$ W1

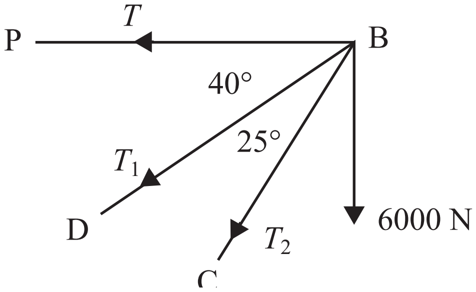
↙ : $T \times \cos 40° + T_1 + 6000 \times \cos 50° = 0$ M1 W1

$T_1 = -T\cos 40° - 6000\cos 50°$

$|T_1| = 9330$ N W1 9

(ii) Alternative solution

At B:



Horizontally: $T + T_1 \cos 40° + T_2 \cos 65° = 0$ (1) M1 W1

Vertically: $6000 + T_1 \sin 40° + T_2 \sin 65° = 0$ (2) MW1

(1) × sin40° :
$T_1 \sin 40° \cos 40° + T_2 \sin 40° \cos 65° = –T \sin 40°$

(2) × cos40° :
$T_1 \sin 40° \cos 40° + T_2 \cos 40° \sin 65° = –6000 \cos 40°$

Subtract: M1
$T_2 (\sin 40° \cos 65° - \cos 40° \sin 65°) = –4596 + 4596 = 0$
$T_2 = 0$ W1

$T_1 \cos 40° = -T$
$|T_1| = 9330$ N MW1

AVAILABLE MARKS

5 **(i)** $E_A = KE = \frac{1}{2}mu^2$ MW1

$E_P = KE + GPE = \frac{1}{2}mv^2 + mg(r - r\cos\theta)$ MW1

$E_P = E_A : \frac{1}{2}mv^2 + mg(r - r\cos\theta) = \frac{1}{2}mu^2$ M1

$v^2 = u^2 - 2gr(1 - \cos\theta)$ W1

(ii) If $v \geqslant 0$ when $\theta = 180°$ then:

$u^2 - 2gr(1 - \cos 180°) \geqslant 0$ M1

$u^2 \geqslant 4gr$

$u_{\min} = 2\sqrt{gr}$ W1

(iii) At B ↓ : $mg + R = \frac{mv^2}{r}$ where R is the reaction M1 W1

$u = 2u_{\min} = 4\sqrt{gr}$ W1

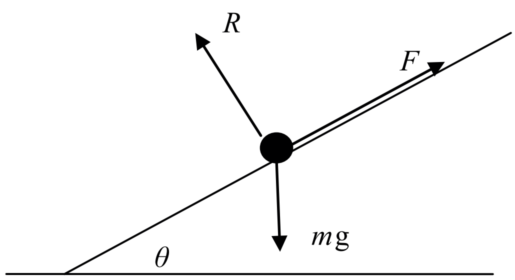
$v^2 = 16gr - 4gr = 12gr$ M1 W1

$$R = -mg + \frac{12mgr}{r}$$ MW1 12

$= 11mg$

AVAILABLE MARKS

6 **(i)**



MW2

m is the mass of the car
R is the normal reaction of the road on the van's tyres.
F is the frictional force between the van's tyres and the road.

(ii) Vertically: $R\cos\theta + F\sin\theta - mg = 0$ (1) M1 W1
Horizontally: $R\sin\theta - F\cos\theta = \frac{mu^2}{r}$ (2) M2 W1

(1) × sinθ: $R\sin\theta\cos\theta + F\sin^2\theta = mg\sin\theta$ M1
(2) × cosθ: $R\sin\theta\cos\theta - F\cos^2\theta = \frac{mu^2}{r}\cos\theta$
Subtract: $F = mg\sin\theta - \frac{mu^2}{r}\cos\theta$ W1

(1) × cosθ: $R\cos^2\theta + F\sin\theta\cos\theta = mg\cos\theta$
(2) × sinθ: $R\sin^2\theta - F\sin\theta\cos\theta = \frac{mu^2}{r}\sin\theta$
Add: $R = \frac{mu^2}{r}\sin\theta + mg\cos\theta$ MW1

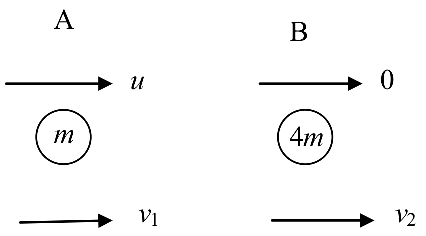
Van is about to slip so $F = \mu R$ M1

$$mg\sin\theta - \frac{mu^2}{r}\cos\theta = \mu(\frac{mu^2}{r}\sin\theta + mg\cos\theta)$$
$$rg\tan\theta - u^2 = \mu u^2\tan\theta + \mu rg$$
$$\tan\theta = \frac{\mu gr + u^2}{gr - \mu u^2}$$ W1

(iii) $$\tan\theta = \frac{0.4 \times 9.8 \times 100 + 15^2}{9.8 \times 100 - 0.4 \times 15^2} = 0.6933$$ M1
$$\theta = 34.7°$$ W1

(iv) Incline is too steep for a normal road. M1 15

7 (i)



AVAILABLE MARKS

$mv_1 + 4mv_2 = mu$ M1 W1

$v_1 - v_2 = -eu$ M1 W1

$5mv_2 = mu + emu$

$v_2 = \frac{1}{5}(1 + e)u$ MW1

$v_1 = v_2 - eu$

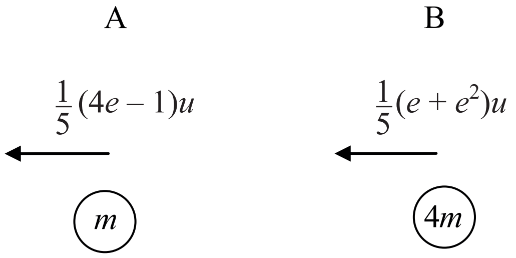
$v_1 = \frac{1}{5}(1 - 4e)u$ MW1

(ii) $v_1 < 0$ so $1 - 4e < 0$ or $e > \frac{1}{4}$ M1 W1

(iii) Let velocity of B after it collides with the wall be v_3

$v_3 = -ev_2 = -\frac{1}{5}(e + e^2)u$ MW1

(iv)



For a further collision:

$\frac{1}{5}(e + e^2) > \frac{1}{5}(4e - 1)$ M1 W1

$e^2 - 3e + 1 > 0$

but $e \leqslant 1$ so $e_{\max} = 0.382$ W1 12

Total **75**