

### Rewarding Learning

#### **ADVANCED**

**General Certificate of Education** 

2016

## **Mathematics**

#### Assessment Unit F2

assessing

Module FP2: Further Pure Mathematics 2



# [AMF21] FRIDAY 17 JUNE, AFTERNOON

#### TIME

1 hour 30 minutes.

#### INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.

Answer all eight questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

You are permitted to use a graphic or scientific calculator in this paper.

#### INFORMATION FOR CANDIDATES

The total mark for this paper is 75

Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.

A copy of the **Mathematical Formulae and Tables booklet** is provided.

Throughout the paper the logarithmic notation used is  $\ln z$  where it is noted that  $\ln z \equiv \log_{a} z$ 

#### Answer all eight questions.

#### Show clearly the full development of your answers.

### Answers should be given to three significant figures unless otherwise stated.

1 (i) Show that

$$\sum_{r=1}^{n} r(r+1) = \frac{n(n+1)(n+2)}{3}$$
 [3]

(ii) Using this result, evaluate

$$\sum_{r=11}^{20} r(r+1) \tag{2}$$

2 (i) Using partial fractions, show that

$$\frac{2x^3 + 6x^2 + 20x + 45}{(x+2)(x^2+9)} \equiv 2 + \frac{1}{x+2} + \frac{x}{x^2+9}$$
 [6]

(ii) Hence or otherwise find the exact value of

$$\int_0^1 \frac{2x^3 + 6x^2 + 20x + 45}{(x+2)(x^2+9)} \, \mathrm{d}x$$
 [5]

3 (a) Find, in radians, the general solution of the equation

$$8\sin\theta + 15\cos\theta = 6 \tag{6}$$

(b) Using small angle approximations, show that when x is small

$$\frac{1+\cos x}{1+\sin\left(\frac{x}{2}\right)}\approx 2-x$$
 [3]

4 (i) Using Maclaurin's theorem, show that a series expansion for  $\ln (1 + 2x)$  up to and including the term in  $x^4$  is

$$\ln\left(1+2x\right) = 2x - 2x^2 + \frac{8x^3}{3} - 4x^4$$
 [6]

(ii) Hence or otherwise find the corresponding Maclaurin's expansion for

$$\ln\left\{\sqrt[3]{\frac{1+2x}{1-2x}}\right\}$$
[3]

5 (i) Show that the equation of the tangent to the parabola  $y^2 = 4ax$  at the point P  $(at^2, 2at)$  is

$$ty = x + at^2 ag{4}$$

[4]

A line perpendicular to this tangent is drawn through the origin. It intersects the tangent at a point Q.

- (ii) As t varies find a Cartesian equation for the locus of Q.
- **6** Prove using mathematical induction that for  $n \ge 1$

$$1^{2} + 4^{2} + 7^{2} + \dots + (3n - 2)^{2} = \frac{n}{2} (6n^{2} - 3n - 1)$$
 [7]

7	The electric current $i$ (amperes) flowing through a coil of inductance $L$ (henrys) and a resistor
	of resistance $R$ (ohms) due to an applied voltage $E$ satisfies the differential equation

$$L\frac{\mathrm{d}i}{\mathrm{d}t} + R \ i = E$$

where L, R and E are positive constants and t is the time in seconds.

- (i) If i = 0 when t = 0, find an expression for i at time t. [8]
- (ii) Describe what happens to *i* as *t* becomes very large. [2]
- 8 (i) Using De Moivre's theorem, show that

$$\sin 5\theta = 16 \sin^5 \theta - 20 \sin^3 \theta + 5 \sin \theta$$
 [6]

(ii) Hence show that if  $\theta$  is not a multiple of  $\pi$ 

$$\frac{\sin 5\theta}{\sin \theta} = 16\cos^4\theta - 12\cos^2\theta + 1$$
 [3]

(iii) By solving the equation  $\sin 5\theta = 0$ , deduce that

$$\cos^2\left(\frac{\pi}{5}\right) = \frac{3+\sqrt{5}}{8} \tag{6}$$

(iv) Write down the corresponding value for  $\cos^2\left(\frac{2\pi}{5}\right)$  [1]

## THIS IS THE END OF THE QUESTION PAPER