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ADVANCED
General Certificate of Education
2016

Mathematics

Assessment Unit F3
assessing
Module FP3: Further Pure Mathematics 3

AMF31

[AMF31]
MONDAY 6 JUNE, AFTERNOON

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all seven** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$, where it is noted that $\ln z \equiv \log_e z$

Answer all seven questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 A triangular prism is sketched in **Fig. 1** below.
O is the origin.
P, Q and T are the points (4, –5, 1), (3, –3, 1) and (4, –4, 16) respectively.

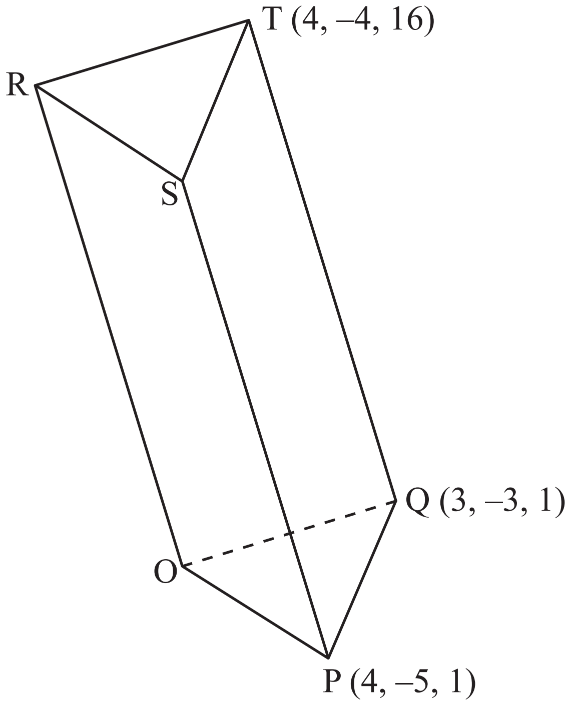

Fig. 1

Using vector methods, find the volume of the prism. [5]

2 Relative to an origin O, points A, B and C have position vectors **a**, **b** and **c** respectively.

(i) Show that a normal to the plane ABC can be written as

$$\mathbf{n} = \mathbf{a} \times \mathbf{b} + \mathbf{b} \times \mathbf{c} + \mathbf{c} \times \mathbf{a}$$ [4]

The plane ABC has equation

$$\mathbf{r}.\,(\mathbf{a} \times \mathbf{b} + \mathbf{b} \times \mathbf{c} + \mathbf{c} \times \mathbf{a}) = d$$

where d is a scalar.

(ii) Find d in terms of **a**, **b** and **c** [3]

(iii) Hence, or otherwise, find the vector equation of the plane through

$$\mathbf{a} = 3\mathbf{i},\ \mathbf{b} = 2\mathbf{j} \text{ and } \mathbf{c} = -\mathbf{k}$$ [3]

3 **(a)** Differentiate

$$\sin\left[\tan^{-1} \frac{1}{\sqrt{x^2 - 1}}\right]$$

with respect to x. [6]

(b) Find

$$\int \frac{\mathrm{d}x}{\sqrt{x - x^2}}$$ [3]

4 **(i)** If

$$I_n = \int x^n \cos x \, dx \qquad n \geqslant 0$$

prove that

$$I_n = x^n \sin x + nx^{n-1} \cos x - n(n-1) I_{n-2} \qquad n \geqslant 2$$ [5]

(ii) Hence find the volume generated by rotating the curve

$$y = x^2 \sqrt{\cos x}$$

through 2π radians, about the x-axis between $x = 0$ and $x = \frac{\pi}{2}$ [6]

5 **(i)** Using the exponential definitions of $\sinh u$ and $\cosh u$, show that

$$2 \sinh^2 u \equiv \cosh 2u - 1$$ [4]

(ii) Using the substitution $2x = \sinh v$, show that

$$\int x^2 \sqrt{1 + 4x^2} \, dx = \frac{1}{256} \sinh [4 \sinh^{-1} (2x)] - \frac{1}{64} \sinh^{-1} (2x) + c$$ [7]

6 **(i)** Sketch the curve

$$y = \text{sech}^{-1}(x) \qquad 0 < x < 1$$ [2]

(ii) Show that

$$\frac{\mathrm{d}}{\mathrm{d}x}\text{sech}^{-1}(x) = \frac{-1}{x\sqrt{1-x^2}}$$ [5]

A machine blade can be modelled as the area bounded by the curve $y = \text{sech}^{-1}(x)$, the x-axis and the ordinates $x = \frac{\sqrt{3}}{2}$ and $x = 1$

(iii) Show that the exact area of this blade is

$$\frac{\pi}{6} - \frac{\sqrt{3}}{2}\text{sech}^{-1}\left(\frac{\sqrt{3}}{2}\right)$$ [5]

(iv) Express this area in the form

$$a\pi + b\ln c$$

where a, b and c are real numbers. [3]

7 The roof of the "House that Jack Built" is lop-sided. It consists of a pair of non-congruent triangles RST and UVW and a pair of non-congruent quadrilaterals RTWU and STWV. PT is a support beam as shown in **Fig. 2** below.

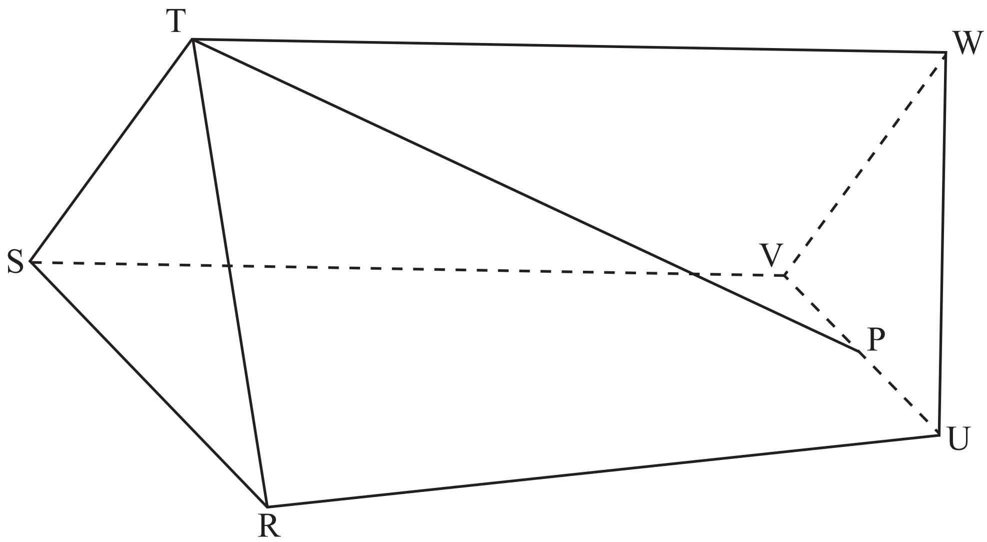

Fig. 2

Relative to an origin,

the line SV has equation $\mathbf{r} = \begin{pmatrix} -6 \\ 5 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix}$,

the line VW has equation $\mathbf{r} = \begin{pmatrix} -2 \\ 11 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -7 \\ 7 \end{pmatrix}$ and

the plane RTWU has equation $21x - 14y + 20z = 84$

(i) Show that the line TW has equation $\mathbf{r} = \begin{pmatrix} 0 \\ 4 \\ 7 \end{pmatrix} + \eta \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix}$ [8]

The line RT has equation

$$\frac{x+2}{-2} = \frac{y+9}{7} = \frac{z}{7}$$

The point P (4, 6, –1) lies on the line VU.

(ii) Find the angle between the plane RTWU and the beam PT. [6]

THIS IS THE END OF THE QUESTION PAPER