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2015

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# Mathematics

Assessment Unit F3  
*assessing*  
Module FP3: Further Pure Mathematics 3



[AMF31]

FRIDAY 5 JUNE, MORNING

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## TIME

1 hour 30 minutes.

## INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.  
Answer **all eight** questions.  
Show clearly the full development of your answers.  
Answers should be given to three significant figures unless otherwise stated.  
You are permitted to use a graphic or scientific calculator in this paper.

## INFORMATION FOR CANDIDATES

The total mark for this paper is 75  
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.  
A copy of the **Mathematical Formulae and Tables booklet** is provided.  
Throughout the paper the logarithmic notation used is  $\ln z$  where it is noted that  $\ln z \equiv \log_e z$

**Answer all eight questions.**

**Show clearly the full development of your answers.**

**Answers should be given to three significant figures unless otherwise stated.**

**1** Find the angle between the vector

$$8\mathbf{i} - \mathbf{j} + 3\mathbf{k}$$

and the normal to the plane

$$5x + 2y - 7z = 17 \quad [4]$$

**2** Find

$$\int \frac{dx}{5 - 4x + 4x^2} \quad [4]$$

**3** Differentiate

$$\tan^{-1}\left(\frac{\sqrt{1-x^2}}{x-3}\right) \quad [6]$$

- 4 (a) Find the intersection of the plane

$$2x - 6y + 5z = 24$$

and the line

$$\frac{x-12}{4} = \frac{y-5}{3} = \frac{z+1}{-5} \quad [4]$$

- (b) Find the volume of the tetrahedron with vertices A (2, 1, 1), B (1, 4, 5), C (0, 2, 3) and D (5, 3, 7). [6]

- 5 (i) Using the exponential definitions of  $\sinh x$  and  $\cosh x$ , show that

$$\tanh^{-1} x = \frac{1}{2} \ln \left( \frac{1+x}{1-x} \right) \quad [4]$$

- (ii) If  $\sinh x - 2 \coth y = 1$

$$\text{and } 2 \sinh x - 5 \coth y = -1$$

find  $x$  and  $y$ , giving your answers in logarithmic form. [6]

- 6 (i) Using the exponential definitions of  $\sinh x$  and  $\cosh x$ , prove the identity

$$\sinh^2 x \equiv \cosh^2 x - 1 \quad [3]$$

$$\text{Let } I_n = \int \cosh^n x \, dx \quad n \geq 0$$

- (ii) Show that

$$I_n = \frac{1}{n} \cosh^{(n-1)} x \sinh x + \frac{n-1}{n} I_{n-2} \quad n \geq 2 \quad [8]$$

- (iii) Find the exact value of

$$\int_0^{\ln 2} \cosh^4 x (1 - \sinh x) \, dx \quad [9]$$

- 7 The pyramid ABCDE has the irregular quadrilateral ABCD for its base as shown in Fig. 1 below.

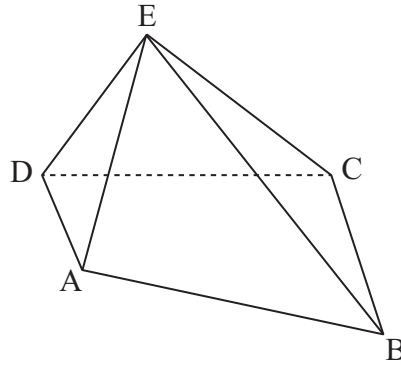


Fig. 1

The line AB is given by  $\{\mathbf{r} - (-6\mathbf{i} - 4\mathbf{j} - 2\mathbf{k})\} \times (10\mathbf{i} - 2\mathbf{j} - \mathbf{k}) = \mathbf{0}$

The line BC is given by  $\{\mathbf{r} - (4\mathbf{i} - 6\mathbf{j} - 3\mathbf{k})\} \times (4\mathbf{i} + 10\mathbf{j} + 5\mathbf{k}) = \mathbf{0}$

The plane CDE is given by  $10x + 18y + 19z = 190$

A, B, C and D lie in the same plane.

Find a vector equation of the line CD.

[9]

- 8 Use integration by parts to find the volume obtained by rotating the curve

$$y = \sin^{-1} x$$

through  $2\pi$  radians about the  $x$ -axis between the ordinates  $x = 0$  and  $x = \frac{1}{2}$

[12]

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**THIS IS THE END OF THE QUESTION PAPER**

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