



Rewarding Learning

ADVANCED
General Certificate of Education
2015

Mathematics

Assessment Unit M2

assessing

Module M2: Mechanics 2

[AMM21]

MONDAY 22 JUNE, AFTERNOON

MARK
SCHEME

GCE Advanced/Advanced Subsidiary (AS) Mathematics

Mark Schemes

Introduction

The mark scheme normally provides the most popular solution to each question. Other solutions given by candidates are evaluated and credit given as appropriate; these alternative methods are not usually illustrated in the published mark scheme.

The marks awarded for each question are shown in the right-hand column and they are prefixed by the letters **M**, **W** and **MW** as appropriate. The key to the mark scheme is given below:

M indicates marks for correct method.

W indicates marks for correct working.

MW indicates marks for combined method and working.

The solution to a question gains marks for correct method and marks for an accurate working based on this method. Where the method is not correct no marks can be given.

A later part of a question may require a candidate to use an answer obtained from an earlier part of the same question. A candidate who gets the wrong answer to the earlier part and goes on to the later part is naturally unaware that the wrong data is being used and is actually undertaking the solution of a parallel problem from the point at which the error occurred. If such a candidate continues to apply correct method, then the candidate's individual working must be followed through from the error. If no further errors are made, then the candidate is penalised only for the initial error. Solutions containing two or more working or transcription errors are treated in the same way. This process is usually referred to as "follow-through marking" and allows a candidate to gain credit for that part of a solution which follows a working or transcription error.

Positive marking:

It is our intention to reward candidates for any demonstration of relevant knowledge, skills or understanding. For this reason we adopt a policy of **following through** their answers, that is, having penalised a candidate for an error, we mark the succeeding parts of the question using the candidate's value or answers and award marks accordingly.

Some common examples of this occur in the following cases:

- (a) a numerical error in one entry in a table of values might lead to several answers being incorrect, but these might not be essentially separate errors;
- (b) readings taken from candidates' inaccurate graphs may not agree with the answers expected but might be consistent with the graphs drawn.

When the candidate misreads a question in such a way as to make the question easier only a proportion of the marks will be available (based on the professional judgement of the examining team).

1 (i) initial position $\Rightarrow t = 0$

$$\mathbf{s} = 2\mathbf{i} - \mathbf{j}$$

M1

no \mathbf{k} vector \therefore on \mathbf{i} - \mathbf{j} plane

W1

(ii) $\mathbf{v} = \frac{d\mathbf{s}}{dt}$

M1

$$\mathbf{v} = (2t)\mathbf{i} + (3t^2)\mathbf{j} + (2t + 2)\mathbf{k}$$

W1

at $t = 2$, $\mathbf{v} = 4\mathbf{i} + 12\mathbf{j} + 6\mathbf{k}$

MW1

$$\text{speed} = \sqrt{4^2 + 12^2 + 6^2}$$

$$\text{speed} = 14 \text{ m s}^{-1}$$

MW1

(iii) $\mathbf{a} = \frac{d\mathbf{v}}{dt}$

M1

$$\mathbf{a} = 2\mathbf{i} + 6t\mathbf{j} + 2\mathbf{k}$$

W1

at $2\mathbf{i} + 8\mathbf{j} + 2\mathbf{k}$,

$$6t = 8$$

$$t = \frac{4}{3}$$

MW1

AVAILABLE
MARKS

9

		AVAILABLE MARKS
2 (i)	unit vector in the direction of \mathbf{F}_1 is $\frac{1}{9} \begin{pmatrix} 4 \\ -4 \\ -7 \end{pmatrix}$	M1
	\mathbf{F}_1 is then $2 \begin{pmatrix} 4 \\ -4 \\ -7 \end{pmatrix}$	W1
	unit vector in the direction of \mathbf{F}_2 is $\frac{1}{3} \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$	
	\mathbf{F}_2 is then $2 \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$	MW1
	Resultant is $\mathbf{F}_1 + \mathbf{F}_2 = \begin{pmatrix} 12 \\ -6 \\ -18 \end{pmatrix}$	MW1
(ii)	Using $\mathbf{F} = m\mathbf{a}$	M1
	$\begin{pmatrix} 12 \\ -6 \\ -18 \end{pmatrix} = 3\mathbf{a}$	
	$\mathbf{a} = \begin{pmatrix} 4 \\ -2 \\ -6 \end{pmatrix}$	W1
(iii)	$\mathbf{v} = \mathbf{u} + \mathbf{at}$	M1
	$\mathbf{v} = \begin{pmatrix} -1 \\ 3 \\ 6 \end{pmatrix} + \begin{pmatrix} 4 \\ -2 \\ -6 \end{pmatrix}$	
	at $t = 1$, $\mathbf{v} = \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}$	MW1
(iv)	scalar product $\begin{pmatrix} -1 \\ 3 \\ 6 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix} = (-1)(3) + (3)(1) + (6)(0) = 0$	M1
	\therefore perpendicular to original direction	MW1
		10

		AVAILABLE MARKS
3 (i)	K.E. = $\frac{1}{2}mv^2$	M1
	Change in K.E. = $\frac{1}{2}(75)7^2 - \frac{1}{2}(75)4^2$	
	Change = 1237.5 J loss	W1
(ii)	Work done = Force \times Distance	M1
	Work done = 14×2100	
	= 29 400 J	W1
(iii)	Loss of P.E. = mgh	M1
	= $75(9.8)(60 - 25)$	
	= 25 725 J	W1
	Total loss of energy = $1237.5 + 25\,725 = 26\,962.5$ J	
	Cyclist's loss of energy provides some of the work needed to overcome resistances – the rest is provided by the cyclist doing work	
	Work done by cyclist = $29\,400 - 26\,962.5$	M1 W1
	Work done by cyclist = 2437.5	
	= 2440 J	W1
		9

4 (i) $(a + bv) \leftarrow \boxed{} \rightarrow \frac{P}{v}$

$P = Fv \quad \therefore \text{Tractive force} = \frac{P}{v}$

Max speed \Rightarrow forces balance

$(a + bv) = \frac{48000}{50}$

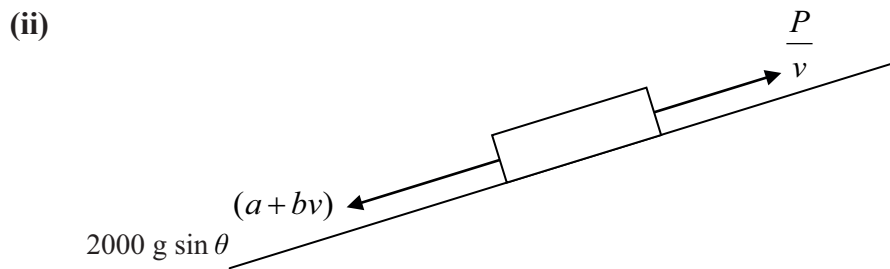
$a + 50b = 960$

M1

M1

W1

W1



Max speed, forces balanced

$(a + b(30)) + 2000g \left(\frac{1}{25}\right) = \frac{48000}{30}$

$a + 30b = 800$

Solving equations simultaneously, $a = 560, b = 8$

M1

MW3

MW1

(iii) $560 + 8(40) \leftarrow \boxed{} \rightarrow \frac{48\,000}{40}$

Resultant = 320 N

$F = ma$

$320 = 2000(a) \quad a = 0.16 \text{ ms}^{-2}$

MW1

M1

W1

AVAILABLE
MARKS

12

			AVAILABLE MARKS	
5 (i)	time to fall 100 m	$s = ut + \frac{1}{2} at^2$	M1	10
		$100 = 0 + \frac{1}{2} (9.8)t^2$		
		$t = 4.5175$	W1	
	horizontally,	$s = ut$		
		$s = 49 \times 4.5175$	M1	
		$s = 221 \text{ m}$	W1	
	(ii) speed vertically	$v = u + at$	M1	
		$v = 0 + 9.8 (4.5175)$		
		$v = 44.27 \text{ m s}^{-1}$	W1	
		Horizontally remains 49 m s^{-1}	M1	
	Speed = $\sqrt{49^2 + 44.27^2} = 66.0 \text{ m s}^{-1}$	M1 W1		
(iii)	logical assumption	MW1		

6 (i) $(v^2 + k^2) \leftarrow \boxed{} \rightarrow 5k^2$

Maximum speed when $v^2 + k^2 = 5k^2$

M1

$$v^2 = 4k^2$$

$$v = 2k$$

W1

(ii) $5k^2 \leftarrow (v^2 + k^2) \leftarrow \boxed{}$

Using $F = ma$

M1

$$-(5k^2 + v^2 + k^2) = Mv \frac{dv}{dx}$$

W1

$$v^2 + 6k^2 = -Mv \frac{dv}{dx}$$

W1

(iii) Separate variables

$$\int \frac{1}{M} dx = \int \frac{-v}{v^2 + 6k^2} dv$$

M2 W1

$$\frac{1}{M} x = -\frac{1}{2} \ln(v^2 + 6k^2) + c$$

W2

$$x = 0, v = 2k$$

M1

$$0 = -\frac{1}{2} \ln((2k)^2 + 6k^2) + c$$

$$c = \frac{1}{2} \ln(10k^2)$$

W1

$$\frac{1}{M} x = \frac{1}{2} \ln(10k^2) - \frac{1}{2} \ln(v^2 + 6k^2) + c$$

$$\frac{1}{M} x = \frac{1}{2} \ln\left(\frac{10k^2}{v^2 + 6k^2}\right)$$

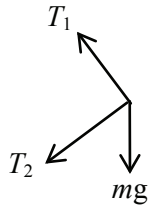
$$\text{At } v = 0 \quad x = \frac{M}{2} \ln \frac{5}{3}$$

MW1

AVAILABLE
MARKS

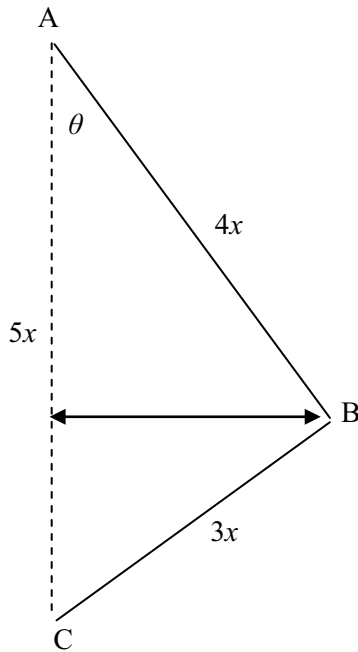
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7 (i)



MW1

(ii)



$$\text{Radius} = 4x \sin \theta$$

M1

$$\sin \theta = \frac{3x}{5x} = \frac{3}{5}$$

$$\begin{aligned} \text{Radius} &= 4x \left(\frac{3}{5} \right) \\ &= \frac{12x}{5} \end{aligned}$$

W1

(iii) Resolving vertically

M1

$$T_1 \cos \theta = T_2 \sin \theta + mg$$

W1

$$T_1 \left(\frac{4}{5} \right) = T_2 \left(\frac{3}{5} \right) + mg$$

MW1

$$4T_1 - 3T_2 = 5mg$$

Resolving horizontally

M1

$$T_1 \sin \theta + T_2 \cos \theta = m r \omega^2$$

W1

M1

$$T_1 \left(\frac{3}{5} \right) + T_2 \left(\frac{4}{5} \right) = m \left(\frac{12x}{5} \right) \left(\frac{5g}{6x} \right)$$

W1

$$3T_1 + 4T_2 = 10mg$$

Solving simultaneously

$$T_1 = \text{tension in AB} = 2mg$$

MW1

$$T_2 = \text{tension in BC} = mg$$

MW1

Total

12

75